202. *Connection of Topological Vector Bundles*

By Akira ASADA

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In this note, we show the notions of connection and curvature can be defined for arbitrary vector bundles (not necessarily the dimension of fibre be finite) over paracompact normal topological space. In usual differential geometry, Nomizu's theorem (Nomizu [4], [5]) and Ambrose-Singer's theorem (Ambrose-Singer [1], Nomizu [5]) show that the connected component of the structure group of a vector boundle is determined by its curvature form. This is also true in our case. (Theorem [2]).

1. *Alexander-Spanier cohomology.* Let X be a paracompact normal topological space. We set

$$\varDelta_s(X)=\{(x, x, \cdots, x)\}\subset \overbrace{X\times X\times \cdots \times X}^{s+1}.$$

By definition, $\varDelta_0(X)=X$.

We denote by L a topological vector space over K, where K is $\boldsymbol{R}$ or $\boldsymbol{C}$. f is an L-valued continuous function on some neighborhood of $\varDelta_s(X)$ in $X\times \cdots \times X$ such that

$$f(x_0, \cdots, x_{s+1})=0, \quad \text{if } x_i=x_{i+1} \text{ for some } i, \quad 0\leq i\leq s.$$

If f and f' are two such functions, then we call f and f' are equivalent if

$$f\mid U(\varDelta_s(X))=f' \mid U(\varDelta_s(X)),$$

where $U(\varDelta_s(X))$ is a neighborhood of $\varDelta_s(X)$. The set of all these equivalence classes is denoted by $C^s(X, L)$ or $C^s(X)$. For simplicity, the class of f is also denoted by f.

We define the homomorphism $d: C^s(X)\to C^{s+1}(X)$ by

$$(df)(x_0, \cdots, x_{s+1})=\sum_{i=0}^{s+1}(-1)^i f(x_0, \cdots, x_{i-1}, x_{i+1}, \cdots, x_{s+1}).$$

Then $d^2=0$. Moreover, setting

$$(k_a f)(x_0, \cdots, x_{s-1})=f(a, x_0, \cdots, x_{s-1}),$$

$k_a^2=0$, and we obtain $(dk_a+k_a d)f=f$ if deg. $f\geq 1$, and $(dk_a+k_a d)f= f-f(a)$ if deg. $f=0$, locally. If we denote ker. $[d: C^s(X)\to C^{s+1}(X)]= F^s(X)$, then we know

$$H^s(X, L)\simeq F^s(X)/dC^{s-1}(X) \quad s\geq 0,$$

where the left hand side is Čech cohomology group. (Godement [3]).

2. *The s-cross-section.* We fix a fibre bundle ξ with base space X, fibre L and structure group G.

We assume that G satisfies following 3-conditions.

(i) The element of G are linear transformations of L.