82. On Axiom Systems of Propositional Calculi. XIX

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(Comm. by Kinjirô KUNUGI, M.J.A., April 12, 1966)

In previous notes (see [1], [2], and [5]), we have proved that equivalential calculus is characterized by each of the axiom systems as follows:

1 $EEpqEqp$, $EEEpqrEpEqr$,

2 $EEpqEEprErq$,

where the fundamental axiom system of equivalential calculus is given by

3 $EEEprEqpErq$, $EEpEqrEEpqr$,

and E is the truth functor in the calculus (see [4]).

First our attempt in this paper is to give a proof of the following

Theorem 1. *The equivalential calculus is characterized by each of the following single axiom systems*:

4 $EEpqEErqEpr$,

5′ $EEpqEErpEqr$,

6′ $EEEpqrEqErp$,

7′ $ErEEqErpEpq$.

Proof. The proofs will be carried out by using the two rules of inference, i.e. the rule of substitution and the rule of detatchment. We shall also use prooflines by J. Lukasiewicz for the proof of theses.

4 p/Epq, $q/EErqEpr$, r/Epq $*C4—C4—5$,

5 $EEpqEpq$.

4 p/Epq, q/Epq $*C5—6$,

6 $EErEpqEEpqr$.

6 p/Erq, q/Epr, r/Epq $*C4—7$,

7 $EEErqEprEpq$.

7 q/p, r/p $*C5$ $q/p—8$,

8 Epp.

4 p/q, r/p $*C8$ $p/q—9$,

9 $EEpqEqp$.

4 $p/EEprEqp$, q/Eqr, r/Erq $*C7$ p/q, q/r, $r/p—C9$ $p/r—10$,

10 $EEEprEqpErq$.

9 $p/EErpEqr$, q/Epq $*C10$ p/r, $r/p—11$,

11 $EEpqEErpEqr$.