98. On Kernels of Invariant Functional Spaces

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Introduction. Deny introduced in [6] the notion of invariant functional spaces and he proved that to an invariant functional space $\mathfrak{X}$ corresponds a convolution kernel κ in the following sense: each potential u_f in $\mathfrak{X}$ generated by a bounded measurable function f with compact support is equal to the convolution $\kappa * f$. In this paper, we shall prove that the converse is valid. That is, for a positive measure κ of positive type, there exists an invariant functional space with kernel κ. Furthermore we shall give a necessary and sufficient condition for a positive measure κ of positive type to be the kernel of a special Dirichlet space.

1. Invariant functional spaces. Let X be a locally compact abelian group. We denote by dx the Haar measure of X. We define two kinds of functional spaces on X.

Definition 1. *A weak invariant functional space $\mathfrak{X}=\mathfrak{X}(X)$ with respect to X and dx is a Hilbert space of real valued locally summable functions satisfying the following two conditions.*

(1.1) *For any compact subset K in X, there exists a positive constant $A(K)$ such that*

$$\left|\int_K u(x)dx\right| \leq A(K)\|u\|$$

for any u in $\mathfrak{X}$.

(1.2) *Let $U_x u$ be a function obtained from u in $\mathfrak{X}$ by the translation $x \in X$. For any u in $\mathfrak{X}$ and any x in X, $U_x u$ is in $\mathfrak{X}$ and $\|U_x u\| = \|u\|$.*

Two functions which are equal *p.p.*[1] in X represent the same element in $\mathfrak{X}$. By the condition (1.1), for any compact subset K in X, there exists an element u_K in $\mathfrak{X}$ such that

$$(u, u_K) = \int_K u(x)dx$$

for any u in $\mathfrak{X}$. Especially when $u_K(x) \geq 0$ *p.p.* in X for any compact subset K, $\mathfrak{X}$ is called a positive weak invariant functional space on X.

Definition 2.[2] *A weak invariant functional space $\mathfrak{X}$ is called*

1) A property is said to hold *p.p.* in a subset E in X if the property holds in E except a set which is locally of measure zero.

2) Cf. [6], p. 12.