170. A Pentavalued Logic and its Algebraic Theory

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In this Note, we shall concern with a pentavalued logic by J. Lukasiewicz and its algebraic theory. The fundamental ideas are due to Professor Gr. C. Moisil (see [1] and [2]).

Let L be a set $\{x, y, z, \cdots\}$ of propositions. The truth values we denote by 0, 1, 2, 3, and 4. We introduce the negation Nx of x by

x	0, 1, 2, 3, 4
Nx	4, 3, 2, 1, 0

The disjunction $\vee$ and the conjunction $\wedge$ are defined as follows:

$\vee$	0	1	2	3	4
0	0	1	2	3	4
1	1	1	2	3	4
2	2	2	2	3	4
3	3	3	3	3	4
4	4	4	4	4	4

$\wedge$	0	1	2	3	4
0	0	0	0	0	0
1	0	1	1	1	1
2	0	1	2	2	2
3	0	1	2	3	3
4	0	1	2	3	4

Then two operations $\vee$, $\wedge$ satisfy the well known axiom of a distributive lattice, hence L is a distributive lattice. On the other hand, consider a commutative ring of characteristic 5, and denote the elements by 0, 1, 2, 3, and 4. Then we have $5x=x+x+x+x+x=0$, and $x^5=xxxxx=x$.

The negation and the modalities have the following algebraic representations.

The negation Nx of x is algebraically denoted by $Nx=4(x+1)$.

The necessity

x	0	1	2	3	4
νx	0	0	0	0	4

and the possibility

x	0	1	2	3	4
μx	0	0	1	1	1

are denoted by $\nu x=x^4+4x^3+x^2+4x$ and $\mu x=3x^4+4x^3+4x^2+4x$ respectively.

In the pentavalued logic, there are two positive modalities as follows:

x	0	1	2	3	4
$N\mu Nx$	0	0	0	4	4
$N\nu Nx$	0	4	4	4	4