157. On J-Groups of Spaces which are Like Projective Planes

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Let K be a CW-complex obtained from attaching a 2n-cell V^{2n} to the n-sphere S^n by a map $f: S^{2n-1} \rightarrow S^n$. We call K a space which is like real, complex, quaternian, Cayley projective plane in accordance with n=1, 2, 4, 8. Our purpose is to calculate J-groups of $K^{(*)}$. Since J-group of a space is determined by its homotopy type we shall use the following notations:

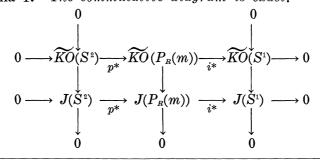
$$\begin{split} P_{R}(m) &= S^{1} \underbrace{f}_{c} e^{2}, \quad (f) \in \pi_{1}(S^{1}) = Z[\iota], \quad (f) = m[\iota] \\ P_{o}(m) &= S^{2} \underbrace{f}_{c} e^{4}, \quad (f) \in \pi_{3}(S^{2}) = Z[h], \quad (f) = m[h] \\ P_{o}(m, n) &= S^{4} \underbrace{f}_{c} e^{8}, \quad (f) \in \pi_{7}(S^{4}) = Z[\nu] + Z_{12}[\tau], \quad (f) = m[\nu] + n[\tau] \\ P_{K}(m, n) &= S^{8} \underbrace{f}_{c} e^{16}, \quad (f) \in \pi_{15}(S^{8}) = Z[\sigma] + Z_{120}[\rho], \quad (f) = m[\sigma] + n[\rho] \\ \text{here } [\iota], \quad [h], \quad [\nu], \quad [\tau], \quad [\sigma], \quad [\rho] \text{ are the generators of respective of the second sec$$

where $[\iota], [h], [\nu], [\tau], [\sigma], [\rho]$ are the generators of respective homotopy groups and $[\iota, \iota_i] = 2[h] + [\tau], [\iota_s, \iota_s] = 2[\sigma] + \rho$.

For example $P_{\mathbb{R}}(2)$, $P_{o}(1)$, $P_{q}(1, 0)$, $P_{\mathbb{K}}(1, 0)$ have respectively the same homotopy type as real, complex, quaternion, Cayley projective planes. Now let $\widetilde{KO}(X)$ denote the abelian group formed by all stable real vector bundles over X. Then there exists the natural onto-homomorphism $J: \widetilde{KO}(X) \rightarrow J(X)$ by the definition of J(X). Hence in order to determine J(X) it is sufficient to calculate $\widetilde{KO}(X)$ and the kernel of J.

1. Case of $P_{\mathbb{R}}(m)$. If *m* is odd we have $\widetilde{KO}(P_{\mathbb{R}}(m))$ is trivial and therefore $J(P_{\mathbb{R}}(m))$ is also trivial. If *m* is even we have $J^{-1}(0)=0$ by the following

Lemma 1. The commutative diagram is exact:



^(*) J. F. Adames: On the group J(X)-1, Topology, Vol. 2 (1963).