189. *On the Absolute Logarithmic Summability of the Allied Series of a Fourier Series*

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1. **Introduction.** § 1.1. **Definition.***) *Let $\lambda=\lambda(w)$ be continuous, differentiable and monotone increasing in* $(0, \infty)$, *and let it tend to infinity as $w\to\infty$. For a given series $\sum_1^\infty a_n$, put*

$$C_r(w)=\sum_{n\leqslant w}\{\lambda(w)-\lambda(n)\}^r a_n \qquad (r\geqslant 0).$$

Then the series $\sum_1^\infty a_n$ is called to be summable $|R, \lambda, r|$ $(r\geqslant 0)$, *if for a positive number A,*

$$\int_A^\infty \left| d\left[\frac{C_r(w)}{\{\lambda(w)\}^r}\right]\right| < \infty.$$

For $r>0$, we have

$$\frac{d}{dw}\left[\frac{C_r(w)}{\{\lambda(w)\}^r}\right]=\frac{r\lambda'(w)}{\{\lambda(w)\}^{1+r}}\sum_{n\leqslant w}\{\lambda(w)-\lambda(n)\}^{r-1}\lambda(n)a_n.$$

Hence $\sum_1^\infty a_n$ is summable $|R, \lambda, r|$ $(r>0)$, if and only if

$$\int_A^\infty \left|\frac{r\lambda'(w)}{\{\lambda(w)\}^{1+r}}\sum_{n\leqslant w}\{\lambda(w)-\lambda(n)\}^{r-1}\lambda(n)a_n\right| dw<\infty.$$

§ 1.2. We suppose that $f(t)$ is integrable in the Lebesgue sense in the interval $(-\pi, \pi)$, and is periodic with period 2π, so that

$$f(t)\sim \tfrac{1}{2}a_0+\sum_1^\infty (a_n \cos nt + b_n \sin nt)=\tfrac{1}{2}a_0+\sum_1^\infty A_n(t).$$

Then the allied series is

$$\sum_1^\infty (b_n \cos nt - a_n \sin nt)=\sum_1^\infty B_n(t).$$

We write

(1) $$\psi(t)=\tfrac{1}{2}\{f(x+t)-f(x-t)\},$$

(2) $$\psi_1(t)=\frac{1}{\log(2\pi/t)}\int_t^\pi \frac{\psi(u)}{u}du.$$

In my thesis [2], I have proved that, if $t^{-1}\psi_1(t)\left(\log\frac{2\pi}{t}\right)^2$ is integrable in $(0, \pi)$, then the allied series of the Fourier series of $f(t)$ is summable $|R, \log w, 2|$. The object of the present paper is to prove the following

Theorem. *If the integral $\int_0^\pi t^{-1}|d\psi_1(t)|$ exists, then the allied*

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*) Mohanty [1].