225. *Integration on Locally Compact Spaces Generated by Positive Linear Functionals Defined on the Space of Continuous Functions with Compact Support and the Riesz Representation Theorem. II*[*)]

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We shall use in this paper the same terminology and notation as in the paper [12].

The main result of § 3 is that the integral functional generated by the Baire volume or the Baire measure gives the smallest extension of the positive linear functional on C_0. It would be interesting to know if representations by means of Borel volumes or of Borel measures are also minimal. In § 5 is given a Baire type characterization of the space of Lebesgue-Bochner measurable functions generated by a positive linear functional defined on C_0. Every function f from the space $M_v(Y)$ of Lebesgue-Bochner measurable functions generated by the Borel or Baire volume v has the property that there exists a function g from the *second Baire class* $C(Y)$ such that $f(x)=g(x)$ v-almost everywhere.

The *classes* $C_n(Y)$ are defined by induction. The class $C_0(Y)$ consists of all continuous functions f with compact support from X into Y. The class $C_n(Y)$ consists of limits of sequences of functions from $C_{n-1}(Y)$.

Let $N(v, Y)$ denote the space of all functions f from the space X into the Banach space Y such that $f(x)=0$ v-almost everywhere.

It follows from the previous result that in every class of the quotient $L(v, Y)/N(v, Y)$ there exists a function such that $g \in C_2(Y)$. Thus if v_1 is a Borel volume and v is the Baire volume being its restriction to the Baire prering then the corresponding quotient spaces are isometrically isomorphic. Moreover the bilinear integral $\int u(f, d\mu)$ considered on the quotient spaces yields the same operator.

This result shows that if one identifies functions equal almost everywhere then the representation by means of Borel volumes or by means of Baire volumes yields essentially the same representation. Since the integration generated by Borel or Baire measure coincides with the integration generated by the corresponding Borel or Baire

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