69. *The Normality of the Product of Two Linearly Ordered Spaces*

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Introduction. Let X and Y be normal topological spaces. The product space $X\times Y$ is not necessarily normal. The problem of deciding when $X\times Y$ is normal, is interesting, in view of the fact that a Hausdorff topological space is normal if and only if any continuous real-valued function defined on any closed subset can be extended to a continuous function on the whole space. In this paper we shall settle this problem in the case where each factor space is a locally compact linearly ordered space. Our result extends the Ball's theorem [1], which assumes that one factor space is compact.

1. By a linearly ordered space we mean a linearly ordered set with the interval topology. It is well known that every such space is normal.

Let L be a non-empty linearly ordered space. An interior gap of L is a Dedekind cut $(A\,|\,B)$ of L such that $A\neq\phi$, $B\neq\phi$. A has no last point and B has no first point. If L has no first (last) point, there exists a left (right) end gap $(\phi\,|\,L)$ $((L\,|\,\phi))$. We denote by L' the set of all gaps of L and by $\bar{L}$ the sum of L and L'. $\bar{L}$ is a compact linearly ordered space. To denote intervals of L, we shall employ the Bourbaki's symbols, $[\ ,\ ]$, $]\leftarrow,[$, etc. Boundaries of an interval of L may be gaps of L as well as points of L.

We define $\rho(L)$ as follows. In case L has a right end gap which is not a limit of interior gaps of L, we put $\rho(L)=\alpha$, where α is a regular initial ordinal such that there exists an increasing sequence $\{x_\lambda;\lambda<\alpha\}$ of points of L which is cofinal with L. In all other cases we put $\rho(L)=0$, more precisely, $\rho(L)=0$ in the following three cases; (1) $L=\phi$, (2) L has a last point, (3) L has a right end gap which is a limit of interior gaps of L.

Let u be any point or gap of L. We put $\rho_-(u)=\rho(\ ]\leftarrow,u[\ )$ and $\rho_+(u)=\rho(\ ]u,\rightarrow[\ ^*)$, where $*$ signifies the inversely ordered set. Finally we define $\tau(L)$ for locally compact linearly ordered space L, as follows. $\tau(L)=$the smallest regular initial ordinal α such that $\rho_-(x)<\alpha$ and $\rho_+(x)<\alpha$ for every point $x\in L$.

We shall say that a point or gap u of L is of type α, if either $\rho_-(u)=\alpha$ or $\rho_+(u)=\alpha$. We denote by ω the first infinite ordinal.