141. On Goursat Problem. I

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1. We shall consider the problem of the unique existence of the solutions in some Gevrey class for the equation written in the following form on $\Omega=\prod_{i=1}^{m}[0, T_i]\times D$ where D is the closure of a bounded domain, in $m+n$ dimensional euclidean space $\prod_{i=1}^{m} R^1_{t_i}\times R^n_x$, i.e. Goursat problem:

$$(1)\qquad \left(\frac{\partial}{\partial t}\right)^{\alpha} u(t, x)=\sum_{\beta,\gamma} a_{\beta\gamma}(t, x)\left(\frac{\partial}{\partial t}\right)^{\beta}\left(\frac{\partial}{\partial x}\right)^{\gamma} u(t, x)+f(t, x)$$

with data

$$(2)\qquad \left(\frac{\partial}{\partial t_i}\right)^{k} u(t, x)\,|_{t_i=0}=\phi_{ik}(t, x) \qquad 0\leqq k\leqq \alpha_i-1 \qquad 1\leqq i\leqq m,$$

where $\phi_{ik}(t, x)$ are defined on $t_i=0$ satisfying

$$(3)\qquad \left(\frac{\partial}{\partial t_i}\right)^{k} \phi_{jl}(t, x)\Big|_{t_i=0}=\left(\frac{\partial}{\partial t_j}\right)^{l} \phi_{ik}(t, x)\Big|_{t_j=0} \qquad i\neq j,\ 1\leqq i, j\leqq m,$$

the notations contained in the above mean

$(t, x)=(t_1, \cdots, t_m, x_1, \cdots, x_n)$,

$\alpha=(\alpha_1, \cdots, \alpha_m)$ multi-positive-integer,

$\beta=(\beta_1, \cdots, \beta_n)$, $\gamma=(\gamma_1, \cdots, \gamma_n)$ multi-nonnegative-integers,

$$\left(\frac{\partial}{\partial t}\right)^{\alpha}=\left(\frac{\partial}{\partial t_1}\right)^{\alpha_1}\cdots\left(\frac{\partial}{\partial t_m}\right)^{\alpha_m}, \qquad \left(\frac{\partial}{\partial x}\right)^{\gamma}=\left(\frac{\partial}{\partial x_1}\right)^{\gamma_1}\cdots\left(\frac{\partial}{\partial x_n}\right)^{\gamma_n},$$

and the summation $\sum_{\beta,\gamma}$ is done for all β, γ satisfying

$$(4)\qquad |\alpha|\geqq|\beta|+|\gamma|, \qquad |\alpha|>|\beta| \text{ and } \alpha_i\geqq\beta_i \qquad 1\leqq i\leqq m,$$

where $|\alpha|=\sum_{i=1}^{m}\alpha_i$ and $|\beta|, |\gamma|$ are similarly defined.

A. Friedman solved the equation with non-linear right hand side under the assumption of the analyticity with respect to t_i variables on $a_{\beta\gamma}(t, x)$ and $f(t, x)$ and a rather stronger condition than (4), [1]. It seems for me that this assumption on t_i variables is essential in his proofs even when we restrict the equation in the linear case. The purpose of this note is to give a remark that we can get a similar result for the linear case under the assumption of the continuity with respect to t_i variables. On this problem Darboux, Goursat, and Bendom treated the case for $m=2$, $\alpha_1=\alpha_2=1$ and a non-linear right hand side, [2]. L. Hörmander solved the case for analytic $a_{\beta\gamma}(t, x)$ and $f(t, x)$ under a weaker condition than