126. *On the Representation of Large Even Integers as Sums of a Prime and an Almost Prime. II*[*)]

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Recently A. A. Buhštab [3, 4] has given a proof for the following[**)]

Theorem. *Every sufficiently large even integer can be represented as a sum of a prime and an almost prime composed of at most three prime factors.*

Here, by an almost prime is meant, in general, an integer>1 with a bounded number of prime factors. His proof of this theorem makes essential use of an important result due to E. Bombieri [1; Theorem 4] (cf. also [7; Theorem 2]) with a complicated combinatorial improvement of the sieve of Eratosthenes, and depends on a long numerical computation for some functions involved therein.

The purpose of the present article is to provide another proof *without* any numerical computation for the theorem stated above.

Almost needless to say, we can also prove that for every fixed integral value of $k\neq 0$ there exist infinitely many primes p such that $p+2k$ has at most three prime factors (cf. [4]).

1. Let k and l be two integers with $k\geqq 1, 0\leqq l<k, (k, l)=1$. Let $\pi(X, k, l)$ denote as usual the number of primes $p\leqq X$ satisfying $p\equiv l \pmod k$. We set

$$R(X, k, l)=\pi(X, k, l)-\frac{\operatorname{li} X}{\phi(k)}$$

and

$$R(X, k)=\max_{(l,k)=1} | R(X, k, l) |,$$

where $\phi(k)$ is the Euler totient function and $\operatorname{li} X$ is the logarithmic integral.

Lemma 1. *For any fixed $\varepsilon>0$ and any constant $A>0$ we have for $Y\leqq X^{1/2-\varepsilon}$*

$$\sum_{m\leqq Y}\tau(m)R(X, m)=O\left(\frac{X}{\log^A X}\right),$$

where $\tau(n)$ denotes the number of divisors of n and where the O-constant may depend on ε and A.

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**) We note that this result was also asserted (without proof) to hold by A. I. Vinogradov [7; Theorem 3] and A. A. Buhštab [4] demonstrated in fact somewhat more.