165. *On Paracompactness and Metrizability of Spaces*

By Yûkiti KATUTA

Ehime University

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1. Introduction. In the previous note [3], we have introduced the notion of an order locally finite collection of subsets of a topological space. This is defined as follows. A collection $\{A_\lambda \mid \lambda \in \Lambda\}$ of subsets of a topological space is called *order locally finite*, if we can introduce a total order $<$ in the index set Λ such that for each $\lambda \in \Lambda$ $\{A_\mu \mid \mu < \lambda\}$ is locally finite at each point of A_λ. It is obvious that every σ-locally finite collection is order locally finite.[1)]

The purpose of this note is prove the following theorems.

Theorem 1. *Let X be a regular space. If there is an order locally finite open covering $\{G_\lambda \mid \lambda \in \Lambda\}$ of X such that for each λ the closure $\bar{G}_\lambda$ of G_λ is paracompact, then X is paracompact.*[2)]

Theorem 2. *Let X be a regular space. If there is an order locally finite open covering $\{G_\lambda \mid \lambda \in \Lambda\}$ of X such that for each λ the boundary $\mathfrak{B}(G_\lambda)$ of G_λ is compact and G_λ (more generally, every closed subset of X contained in G_λ) is paracompact, then X is paracompact.*

Theorem 3. *Let X be a collectionwise normal T_1-space. If there is an order locally finite open covering $\{G_\lambda \mid \lambda \in \Lambda\}$ of X such that for each λ the boundary $\mathfrak{B}(G_\lambda)$ of G_λ is paracompact and G_λ (more generally, every closed subset of X contained in G_λ) is paracompact, then X is paracompact.*

Theorem 4. *Let X be a collectionwise normal T_1-space. If there are a closed covering $\{F_\lambda \mid \lambda \in \Lambda\}$ and an order locally finite open covering $\{G_\lambda \mid \lambda \in \Lambda\}$ of X such that for each λ $F_\lambda \subset G_\lambda$ and F_λ is paracompact, then X is paracompact.*

Applying the metrization theorem of J. Nagata [6] and Yu. M. Smirnov [7] that a space which is the union of a locally finite collection of closed metrizable subsets is metrizable, from Theorems 1, 2, and 3 we obtain immediately the following Theorems 5, 6, and 7 respectively.

Theorem 5. *Let X be a regular space. If there is an order*

1) H. Tamano [9] has introduced the notion of *linearly locally finite* collections. By definition, every σ-locally finite collection is linearly locally finite and every linearly locally finite collection is order locally finite (but not conversely).

2) This theorem has been proved by Tamano [9] in the case when X is a completely regular T_1-space and $\{G_\lambda \mid \lambda \in \Lambda\}$ is lineary locally finite.