153. *On the Principle of Limiting Amplitude*

By Nobuhisa IWASAKI

(Comm. by Kinjirô KUNUGI, M.J.A., Oct. 12, 1967)

§ 1. **Introduction and Theorem.** We study the behavior for large time of solutions of wave equations with a harmonic forcing term in the three dimensional euclidian space. That is called the principle of limiting amplitude. This principle states that every solution $u(x_1 t)$ for the initial value problem,

$$\left\{\frac{\partial^2}{\partial t^2}+b(x)\frac{\partial}{\partial t}-\Delta+c(x)\right\}u(x, t)=f(x)e^{i\omega t} \tag{1.1}$$

$$u(x, t)\Big|_{t=0}=\frac{\partial}{\partial x}u(x, t)\Big|_{t=0}=0, \tag{1.2}$$

tends to the steady state solution, $e^{i\omega t}v(x, i\omega)$, uniformly on bounded sets at $t\to\infty$. There $v(x, i\omega)$ satisfies the elliptic equation.

$$\{-\Delta+c(x)+i\omega b(x)-\omega^2\}V(x, i\omega)=f(x), \tag{1.3}$$

and the Sommerfeld radiation conditions at infinity. In the case when $b(x)\equiv 0$ and the real valued function $c(x)$ is once continuously differentiable and its support is compact, this principle has been proved by D. A. Ladyzenskaja [1]. Here the rate of approach to steady state is like $e^{-\varepsilon t}$, $\exists\varepsilon>\sigma$, as $t\to\infty$. When $b(x)$ and $c(x)$ satisfy that $b(x)\geq 0$, $b(x)=\frac{1}{|x|^{3+\varepsilon}}$, $c(x)=\frac{1}{|x|^{2+\varepsilon}}$ as $|x|\to\infty$, and others, S. Mizohata and K. Mochizuki [2] has shown the principle, but they did not give the rate of approach. In this paper, we shall obtain the rate under the assumption that the real-valued function $b(x)\geq 0$, $c(x)\geq 0$ are bounded and their supports are compact.

Theorem. *Let $f(x)$ $b(x)$, and $c(x)$ be funtions which satisfy the following conditions.*

i) *$f(x)$, $b(x)$, and $c(x)$ vanish outside a bounded set*

ii) $\sum_{|\alpha|\leq 2}|D^\alpha f| \quad \in L^2(E^3)$

iii) *$b(x)\geq 0$, $c(x)\geq 0$, and they are bounded functions.*

And let $u(x, t)$ be a solution for initial value problem (1.1), (1.2).

Then there exists a steady sate $e^{-i\omega t}V(x)$, such that

$$\max_{x\in k}|u(x, t)-V(x)e^{i\omega t}|\leq C\cdot e^{-\varepsilon t},\ \exists\varepsilon>0 \text{ as } t\to\infty, \tag{1.4}$$

and V is a solution (1.3) *satisfying the Sommerfeld radiation conditions at infinity. Here K is a bounded set of E^3.* We can regard a solution $u(x, t)$ as a twice continuously differentiable function $u(t)$ from $[0, \infty)$ to $L^2(E^3)$ and as a continuous function to $\varepsilon^2_{L^2}(E^3)$. In this sense there exists the unique solution of (1.1), (1.2) if