184. *Notes on Groupoids and their Automorphism Groups*

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A groupoid is a set with a binary operation which need not be associative. The group of all automorphisms of a groupoid G is called the automorphism group of G and it is denoted by $\mathfrak{A}(G)$. Let $\mathfrak{S}(G)$ denote the symmetric group on the set G. In [2] the author determined the structure of G satisfying $\mathfrak{A}(G)=\mathfrak{S}(G)$. This paper supplements equivalent conditions to the theorem in case $|G|>4$, and adds some related results.

In [2] the author gave the following theorem.

Theorem 1. *Let G be a groupoid. $\mathfrak{A}(G)=\mathfrak{S}(G)$ if and only if G is either isomorphic or anti-isomorphic onto one on the following types*:

(1.1) *A right zero semigroup*: $xy=y$ *for all* x, y.

(1.2) *The idempotent quasigroup of order* 3.

(1.3) *The groupoid* $\{1, 2\}$ *of order* 2 *defined by*

$$x\cdot 1=2,\ x\cdot 2=1 \quad \textit{for } x=1, 2.$$

Before introducing the main theorem in this paper, we mention some remarks on the terminology (see [1]). We do not assume the finiteness of G.

By a finite permutation φ of a set G we mean a permutation φ of G such that the set $\{x \in G; x\varphi \neq x\}$ is finite. A permutation φ of G is called even if and only if φ is a finite permutation which is the product of even number of substitutions (i.e. cycles of length 2). An odd permutation is defined in a similar way. Let $\mathfrak{H}$ be a permutation group on G. Let k be a positive integer with $k \leqq |G|$. $\mathfrak{H}$ is called k-ply transitive if and only if for an arbitrary set of k distinct elements $a_1, \cdots, a_k$ and for an arbitrary set of k distinct elements $a'_1, \cdots, a'_k$, there is $\varphi \in \mathfrak{H}$ such that $a_i\varphi = a'_i$ for $i=1, \cdots, k$. Let $\mathfrak{B}(G)$ denote the group of all automorphisms and all anti-automorphisms of G. $\mathfrak{A}(G)$ is a subgroup of $\mathfrak{B}(G)$ and the index of $\mathfrak{A}(G)$ in $\mathfrak{B}(G)$ is 2. Let $\mathfrak{S}^*(G)$ denote the group of all finite permutations of G.

Theorem 2. *Let G be a groupoid with $|G|>4$. Then the following statements are equivalent.*

(2.1) A groupoid G is isomorphic onto either a right zero