210. *On Some Classes of Operators. II*

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In [2], [4] some classes of non-normal operators were introduced namely the classes $C(N, k)$. The definition of these classes is:

Definition 1. *An operator T on Hilbert space is in $C(N, k)$ if*

$$\|Tx\|^k \leq \|T^k x\|$$

for every unit vector $x \in H$.

The aim of this Note is to give some new results connected with these classes.

Theorem 1. *There exists an operator T which is normaloid and is not in $C(N, k)$ for all k.*

Proof. We consider the operator [6]

$$A=\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$

and I be the one-dimensional identity operator and put

$$T=A \oplus I.$$

It is very easy to see (this was firstly observed by Toeplitz) that

$$\mathrm{Cl}\, W(A)=\{z, |z| \leqslant 1/2\}$$

(Here $W(A)=\{\langle Ax, x\rangle, \|x\|=1\}$ and $\mathrm{Cl}\, E$ denotes the closure of such a set E). Also $\mathrm{Cl}\, W(T)$ is the covex hull of $\mathrm{Cl}\, W(A)$ and the point $\{1\}$. Since

$$\sup_{\|x\|=1} |\langle Tx, x\rangle| \geqslant 1=\|T\|$$

it is clear that T is normaloid.

We consider T as an operator on a finite dimensional space. Then it is clear that if $T \in C(N, k)$, T must be normal by Theorem 3 of [4].

This leads to the fact that T is not in $C(N, k)$ for all k and the theorem is proved.

Remark 1. The construction of examples of operators which are normaloid and are not in $C(N, k)$ for all k has the following reason: the restriction of a normaloid operator to an invariant subspace is not generally normaloid.

The following theorem represents a generalization to our case of results in [10], [11].

Theorem 2. *If $p(\lambda)$ is a polynomial non-vanishing on $\sigma(T)-\{0\}$ and $p(T)$ is a Riesz operator of class $C(N, k)$ for some k then T is normal.*

Proof. The fact that $p(\lambda)$ is non-vanishing on $\sigma(T)-\{0\}$ implies [9] that T is a Riesz-operator. The proof of the fact that T must be normal is modeled on a proof of Theorem 2 of [3]. By Theorem