208. *An Abstract Integral*

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1. Introduction. In the previous paper [2] the author considered an abstract treatment of derivatives and integrals of Perron's type. S. Izumi [1] has given an abstract consideration of the general and special Denjoy integrals using the lemma of P. Romanovski.

The aim of this paper is to extend and modify Izumi's idea and to obtain a more comphrehensive abstract integral which contains the approximately continuous Denjoy integral (AD-integral) defined by the author [3] as a special case.

2. Derivative and absolute continuity in abstract sense. Let $f(x)$ be a real valued function defined on the interval $I=[a, b]$ and α, β be real constants. We call an operation $abDf(x)$ *abstract derivative* of $f(x)$ at x provided that

(i) if $f(x)$ is differentiable at x in the ordinary sense then

$$abDf(x)=f'(x);$$

(ii) $\quad abD(\alpha f(x)+\beta g(x))=\alpha abDf(x)+\beta abDg(x).$

A real valued function $F(x)$ is said to be *absolutely continuous in abstract sense* on the set E, written by $F\in ab\,AC_E$, if the following conditions are satisfied.

(iii) If $F\in ab\,AC_E$ and $E'\subset E$ then $F\in ab\,AC_{E'}$.

(iv) If $F, G\in ab\,AC_E$ then $\alpha F+\beta G\in ab\,AC_E$.

(v) If F is absolutely continuous in the ordinary sense on E then $F\in ab\,AC_E$.

(vi) If $F\in ab\,AC_E$ and E is closed then $abDF(x)$ exists at almost all points of E.

(vii) If $F(x)$ is approximately continuous on (a, b) and is non-decreasing on each complementary interval of closed set E with respect to (a, b) and if $F\in ab\,AC_E$ and $abDF(x)\geqq 0$ a.e. on E then $F(x)$ is non-decreasing on (a, b).

A finite function $F(x)$ is said to be *generalized absolutely continuous in abstract sense* on $I=[a, b]$, symbolically $F\in ab\,ACG_I$, if the interval I is the sum of a countable number of closed sets E_k $(k=1, 2\cdots)$ such that $F\in ab\,AC_{E_k}$.

3. Abstract integral. Lemma 1. *If a non-void closed set E is the sum of a countable number of closed sets E_k $(k=1, 2\cdots)$, then there exists an interval (l, m) containing points of E and an*