3. *An Extension of Beurling's Theorem. II*

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This article is the continuation of the previous paper of the same title. We shall prove the following

Lemma 5. *Let F be a set of positive capacity in B and suppose for any point $p\in F\cap B_1^N$ there exists a contact set $\Delta(p)$ of p such that $\lim\limits_{n=\infty}{}_{\Delta(p)\cap v_n(p)}N(z,p)>0$. Let $\{G_n\}$ be a decreasing sequence of domains in $R-R_0$ such that $G_n\supset(G_{n,p}\cap v_m(p)\cap\Delta(p))$, where $G_{n,p}\overset{N}{\ni} p$ and m and $G_{n,p}$ depend on p and G_n. Then*

$$\omega(\{G_n\},z)>0.$$

Put $\Gamma_m=E\left[p\in B_1^N\colon \int\limits_{\partial R_0}\frac{\partial}{\partial n}\lim\limits_{n=\infty}{}_{G_n',p\cap v_n(p)\cap\Delta(p)}N(z,p)ds\geqq\frac{2\pi}{m}\right]$. Then by Lemma 1.3) $F=B_0^N+\sum\limits_{m=1}^{\infty}\Gamma_m$. Now since B_0^N is an F_σ set of capacity zero, there exists a number l_0 and a closed set F' of positive capacity in F such $(F'\cap B_1^N)\subset\Gamma_{l_0}$. Hence there exists a positive mass distribution μ on $F'\cap B_1^N$ such that $V(z)=\int N(z,p)d\mu(p)$ and $V(z)\leqq 1$ in $R-R_0$. ${}_{G_n}V(z)={}_{G_n}\left(\int N(z,p)d\mu(p)\right)\geqq\int\lim\limits_{n=\infty}{}_{G_n',p\cap\Delta(p)\cap v_n(p)}N(z,p)d\mu(p)$ $=\int\lim\limits_{n=\infty}{}_{\Delta(p)\cap v_n(p)}N(z,p)d\mu(p)$ for any n and $\int\limits_{\partial R_0}\frac{\partial}{\partial n}{}_{G_n}V(z)ds\geqq\frac{2\pi}{l_0}\int d\mu(p)>0$. Let $\omega_n(z)=\omega(G_n,z,R-R_0)$. Then by the maximum principle $\omega_n(z)\geqq{}_{G_n}V(z)$. Let $n\to\infty$. Then

$$\omega(\{G_n\},z)\geqq\lim_{n=\infty}{}_{G_n}V(z)>0.$$

Let $w=f(z)\colon z\in R$ be an analytic function whose values fall on a basic surface $\underline{R}$. Suppose N-Martin's topology is defined in $\bar{R}-R_0$. Let $p\in B_1^N$ and let $\Delta(p)$ be a contact set of $p\in B_1^N$. Put

$$M(f(p))=\bigcap_\tau\overline{f(G_\tau)}\ \text{and}\ \Delta(f(p))=\bigcap_\tau\overline{f(\Delta(p)\cap G_\tau)}.$$

Then $M(f(p))\subset\Delta(f(p))$, where $\{G_\tau\}$ runs over all domains G_τ such that $G_\tau\overset{N}{\ni}p$ and the closure is taken with respect to the topology of $\underline{R}$.

Let $\mathfrak{F}$ be a closed set in R. We suppose $\mathfrak{F}$ is contained in a local parameter disc $|w|<1$ and let $A(r)$ be the area of R (not of R) on $E[w\colon \mathrm{dist}\,(w,\mathfrak{F})\leqq r]=\mathfrak{F}_r$. We suppose $\mathfrak{F}$ in $|w|<\frac{1}{2}$. If $\mathfrak{F}$ is one point a and $\varliminf\limits_{r\to 0}\frac{A(r)}{r^2}<\infty$, a is called an ordinary point. A. Beurling and M. Tsuji proved the following