51. On a New Positive Linear Polynomial Operator

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In this note we introduce a new positive linear polynomial operator: $P_m^{[\alpha]}(f;x)=P_m^{[\alpha]}(f(t);x)$, corresponding to a function $f=f(x)$, defined on the interval $[0,1]$, and to a parameter $\alpha\geqq 0$, which may depend only on the natural number m. This operator is

$$(1)\qquad P_m^{[\alpha]}(f;x)=\sum_{k=0}^{m} w_{m,k}(x;\alpha)f\left(\frac{k}{m}\right),$$

where

$$w_{m,k}(x;\alpha)$$
$$=\binom{m}{k}\frac{x(x+\alpha)\cdots(x+\overline{k-1}\alpha)(1-x)(1-x+\alpha)\cdots(1-x+\overline{m-k-1}\alpha)}{(1+\alpha)(1+2\alpha)\cdots(1+\overline{m-1}\alpha)}.$$

One observes that it represents a *polynomial* of *degree* m.

Because $\alpha\geqq 0$, we have $w_{m,k}(x;\alpha)\geqq 0$ for $x\in[0,1]$. Therefore the linear (additive and homogeneous) operator (1) is *positive* on the interval $[0,1]$. In fact we have here a class of operators depending on the parameter α.

First we should remark that for $\alpha=0$ the operator (1) reduces to the Bernstein polynomial

$$(2)\qquad B_m(f;x)=\sum_{k=0}^{m} p_{m,k}(x)f\left(\frac{k}{m}\right),\quad p_{m,k}(x)=\binom{m}{k}x^k(1-x)^{m-k}.$$

Then we wish to make the remark that if we choose $\alpha=0(m^{-1})$ and use the change of variable $x=\frac{n}{m}y$, n being a natural number not depending on m, then—denoting again the variable by x—we obtain from our operator the Mirakyan operator

$$(3)\qquad M_n(f;x)=e^{-nx}\sum_{k=0}^{\infty}\frac{(nx)^k}{k!}f\left(\frac{k}{n}\right).$$

After these preliminaries we can state several theorems, the proofs of which will appear in the journal: Rev. Roumaine Math. Pures Appl.

Theorem 1. *If the parameter* α *has a fixed non-negative value in each term of the sequence* $\{P_m^{[\alpha]}(f;x)\}$, *then there exists the following relationship*