127. *Π-imbeddings of Homotopy Spheres*

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1. Let Θ_n be the group of homotopy n-spheres and let us consider the π-imbedding (i.e., imbedding with a trivial normal bundle) of $\tilde{S}^n \in \Theta_n$ in the $(n+k)$-dimensional euclidean space R^{n+k}.

If $\tilde{S}^n$ is π-imbeddable in R^{n+k}, then the connected sum $\tilde{S}^n \# \tilde{S}^n$ is also π-imbeddable in R^{n+k}. Thus we want to determine the smallest codimension with which the generator $\tilde{S}^n_0$ of Θ_n is π-imbeddable.

In [1], W. C. Hsiang, J. Levine, and R. H. Szczarba showed that every homotopy n-sphere is π-imbeddable in R^{n+k} if $k \geqq n-2$ for all n or $k > \frac{n+1}{2}$ for $n \leqq 15$. They also showed that $\tilde{S}^{16}_0 (\in \Theta_{16} \cong Z_2)$ is not π-imbeddable in R^{29}.

Now, owing to the classification theorem of S. P. Novikov [6], we can calculate the number of the differentiable structures of a direct product of spheres and this gives us some informations on our problem. In this way, we obtain the following results.

n	8	9	10	13	14	15	16	17
order of Θ_n	2	8	6	3	2	16256	2	16
order of $\Theta_n(\partial\pi)$	1	2	1	1	1	8128	1	2
k	4	4	4∼6	3∼4	?∼8	3∼4	14	?∼13

(k is the smallest codimension with which the generator of Θ_n is π-imbeddable.)

If $\Theta_n = 0$ or $\Theta_n(\partial\pi)$, then $k=1$ or 2 respectively ([3], Theorem I).

2. Lemma 1. *If $\tilde{S}^n (n \geqq 5)$ is π-imbeddable in R^{n+k}, then $\tilde{S}^n \times S^{k-1}$ is diffeomorphic to $S^n \times S^{k-1}$.*

The proof is the same as that of Theorem 5.2 in [7].

Conversely, we have

Lemma 2. *If $\tilde{S}^n \times S^{k-1}$ and $S^n \times S^{k-1}$ are diffeomorphic modulo a point, then $\tilde{S}^n$ is π-imbeddable in R^{n+k}.*

Proof. Since $(\tilde{S}^n \times S^{k-1}) \# \tilde{S}^{n+k-1}$ is diffeomorphic to $S^n \times S^{k-1}$ and $\tilde{S}^{n+k-1}$ can be summed to $\tilde{S}^n \times S^{k-1}$ away from $\tilde{S}^n \times x_0$ for some point $x_0 \in S^{k-1}$, the imbedding

$$\tilde{S}^n \subset (\tilde{S}^n \times S^{k-1}) \# \tilde{S}^{n+k-1} \approx S^n \times S^{k-1} \subset R^{n+k}$$

has a trivial normal bundle.