173. An Axiomatic Characterization of the Large Inductive Dimension for Metric Spaces

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An axiomatic characterization of dimension was given by Menger for subsets of Euclidean plane in 1929 [1]. Recently, Nishiura generalized Menger's result in the following form [4]:

Suppose that f is an extended real-valued function on the collection of separable metrizable spaces. Then f is the dimension function if and only if f satisfies the following seven conditions:

(a) *f is topological*; that is, X homeomorphic to Y implies $f(X)=f(Y)$.

(b) *f is monotone*; that is, $X\subset Y$ implies $f(X)\leqq f(Y)$.

(c) *f is F_σ-constant*; that is, $X=\bigcup_{i=1}^{\infty} X_i$, where X_i is a closed subset of X, $i=1, 2, \cdots$, implies $f(X)\leqq \sup f(X_i)$.

(d) *f is inductively subadditive*; that is, $X=A\cup B$ implies $f(X)\leqq f(A)+f(B)+1$.

(e) *f is compactifiable*; that is, each space X is homeomorphic to a subspace of a compact space Y for which $f(X)=f(Y)$.

(f) *f is pseudo-inductive*; that is, for each space X and $x\in X$ there are arbitrarily small neighbourhoods U of x such that $f(\bar{U}-U)\leqq f(X)-1$. (We agree that $\infty-1=\infty$.)

(g) $f(\{\phi\})=0$.

Furthermore, the seven conditions are independent.

The purpose of the present paper is to give a generalization of Nishiura's result for metrizable spaces without separability. For any metric space X we have $\operatorname{Ind} X=\dim X$ (cf. [2] or [3]), but there exists a complete metric space X with $\operatorname{ind} X=0$ and $\operatorname{Ind} X=1$ [5]. In view of the fact that the large inductive dimension has many remarkable properties besides the property mentioned above, this paper will be devoted to the study of large inductive dimensions.

Before stating the theorem, we fix some terminology. For the definitions of ind (the small inductive dimension), Ind (the large inductive dimension) and dim (the covering dimension), see [3]. Spaces $X, Y, \cdots$ will be assumed to be metrizable spaces, and dim will be used both for the large inductive dimension and for the covering dimension. Let Ω be a non-empty set. Generalized Baire space $N(\Omega)$