24. On $\mathfrak{R}$-convex Sets in a Topological $\mathfrak{R}$-space

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§1. **Introduction.** In this paper we shall consider the Krein-Milman's Theorem and the applications on a topological $\mathfrak{R}$-space which has not vector spacestructure. The notion of topological $\mathfrak{R}$-spaces is introduced by E. Deák [1]-[5].

We shall first give the some definitions.

(1.1) A system R of the ordered pair (G, F) consisting of the subsets of a nonempty set X is called a *Richtung* of X, if it satisfies the following conditions:

(R_1) $(\phi, \phi), (X, X) \in R$.

(R_2) For any $(G, F) \in R$, $G \subseteqq F$ and for two different pairs $(G_1, F_1), (G_2, F_2) \in R$, $F_1 \subseteqq G_2$ or $F_2 \subseteqq G_1$.

(R_3) Let $\mathcal{G}(R)$ be a family of the first part of all elements of R. $\bigcup \{G \mid G \in \mathcal{G}^*\} \in \mathcal{G}(R)$ $(\mathcal{G}^* \subset \mathcal{G}(R), \mathcal{G}^* \neq \phi)$.

(R_4) Let $\mathcal{F}(R)$ be a family of the second part of all elements of R. $\bigcap \{F \mid F \in \mathcal{F}^*\} \in \mathcal{F}(R)$ $(\mathcal{F}^* \subset \mathcal{F}(R), \mathcal{F}^* \neq \phi)$.

(R_5) $\bigcup \{F - G \mid (G, F) \in R\} = X$.

(1.2) Let $\mathfrak{R} = \{R_\alpha \mid R_\alpha$: Richtung, $\alpha \in A\}$. A $\mathfrak{R}$-space is an ordered pair $(X, \mathfrak{R})$ such that the following separation axiom is satisfied.

(S.A) Any set of the type, $\bigcap \{F_\alpha - G_\alpha \mid (G_\alpha, F_\alpha) \in R, \alpha \in A\}$ contains at most one element.

(1.3) For a $\mathfrak{R}$-space $(X, \mathfrak{R})$, the set G, $X - F$ or F, $X - G$ is called the open or closed $\mathfrak{R}$-half spaces of X.

(1.4) A $\mathfrak{R}$-space $(X, \mathfrak{R})$ is called a *topological $\mathfrak{R}$-space* if we introduce the topology in X such that a family of all open $\mathfrak{R}$-half spaces is a subbasis.

(1.5) For any Richtung R of X, it is clear that the relation:

$(G_1, F_1) \prec (G_2, F_2) \Longleftrightarrow F_1 \subseteqq G_2$ is a linear order of R.

For any $G \in \mathcal{G}(R)$ or $F \in \mathcal{F}(R)$ is the first or second part of at most two different elements of R. $G(R : F)$ or $\bar{G}(R : F)$ denotes the smaller or larger set $G \in \mathcal{G}(R)$ such that $(G, F) \in R$, and in the same way we can define $F(R : G)$ and $\bar{F}(R : G)$ for any $G \in \mathcal{G}(R)$.

(1.6) For any nonempty set $E \subset X$ and $R \in \mathfrak{R}$ of a $\mathfrak{R}$-space $(X, \mathfrak{R})$, we give the following notations:

$$G_E(R) = \bigcup \{G \in \mathcal{G}(R) \mid G \cap E = \phi\},$$
$$F_E(R) = \bigcap \{F \in \mathcal{F}(R) \mid F \supseteqq E\},$$
$$G_x(R) = \bigcup \{G \in \mathcal{G}(R) \mid G \not\ni x\},$$