57. *Leśniewski's Protothetics S1, S2. II*

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In this paper, finally we shall prove that every theorem of S2 is a theorem of S1.

For further theorems we shall use the following abbreviations:

instead of $[p]\{p\equiv p\}$ we shall write 1,
instead of $[p]\{p\}$ we shall write 0.

The abbreviation of A1 has the form:

A1′ $[f, q]\{f(1)\supset(f(0)\supset f(q))\}$.

Lemma 5. *The proposition*

(12) $[p, q, r]\{(p\equiv q)\equiv((r\equiv q)\equiv(p\equiv r))\}$

is a theorem of S1.

Proof. We shall prove further theorems of S1.

T27	$1\supset 1$	
T28	$0\supset 0$	
T29	$1\equiv 1$	(T10; T27; def (2), ii)
T30	$0\equiv 0$	(T10; T28; def (2), ii)
T31	$(0\equiv 1)\supset(1\equiv 0)$	

Proof.

(1)	$(0\equiv 1)\supset$	
(2)	$\sim((0\supset 1)\supset\sim(1\supset 0))$	(def (2), i; 1)
(3)	$1\supset 0$	(T9; 2)
(4)	$0\supset 1$	(T8; 2)
(5)	$\sim((1\supset 0)\supset\sim(0\supset 1))$	(T10; 3; 4)
(6)	$1\equiv 0$	(def (2), ii; 5)

T32	$(1\equiv 0)\supset(0\equiv 1)$	(likewise T31)
T33	$(0\equiv 1)\equiv(1\equiv 0)$	(T10; T31; T32; def (2), ii)
T34	$(1\equiv 0)\equiv(0\equiv 1)$	(T10; T32; T31; def (2), ii)
T35	$(1\equiv 1)\equiv(1\equiv 1)$	(T10; T29; def (2), ii)
T36	$(0\equiv 0)\equiv(0\equiv 0)$	(T10; T30; def (2), ii)
T37	$(1\equiv 1)\equiv((0\equiv 1)\equiv(1\equiv 0))$	(T10; T29; T33; def (2), ii)
T38	$(1\equiv 1)\equiv((1\equiv 1)\equiv(1\equiv 1))$	(T10; T29; T35; def (2), ii)

From the fact that Lemma 2 is true, we may introduce the following definition.

D5	$[p]\{\varphi_1(p)\equiv((1\equiv 1)\equiv((p\equiv 1)\equiv(1\equiv p)))\}$	
T39	$\varphi_1(0)$	(T37; D5)
T40	$\varphi_1(1)$	(T38; D5)
T41	$[r]\{\varphi_1(r)\}$	(A1′; T40; T39)