53. *On Ranked Spaces and Linearity. II*

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In this note we shall give a definition of linear ranked spaces, axioms of which are weaker than those given in [2]. Sometimes this definition is more convenient to use, in particular, to study for the notions connected with fundamental sequences of neighbourhoods. Hereafter we shall treat only ranked spaces with indicator ω_0[1]. Throughout this note, $x, y, \cdots$ will denote points of a ranked space, $\mathfrak{V}_n(x)$ the system of neighbourhoods of x with rank n, $\{u_n(x)\}$, $\{v_n(x)\}$, $\cdots$ fundamental sequences of neighbourhoods with respect to x.

§1. Definition of linear ranked spaces. Let E be a ranked space, and also a linear space over real or complex field. We call E a linear ranked space, if linear operations in E are continuous in the following sense:

(I) For any $\{u_n(x)\}$ and $\{v_n(y)\}$, there is a $\{w_n(x+y)\}$ such that $u_n(x)+v_n(y)\subseteq w_n(x+y)$.

(II) For any $\{u_n(x)\}$ and $\{\lambda_n\}$ with $\lim \lambda_n=\lambda$, there is a $\{v_n(\lambda x)\}$ such that $\lambda_n u_n(x)\subseteq v_n(\lambda x)$.

(I) implies the continuity of addition; if $\{\lim x_n\}\ni x$ and $\{\lim y_n\}\ni y$, then $\{\lim (x_n+y_n)\}\ni x+y$, and (II), the continuity of scalar multiplication; if $\{\lim x_n\}\ni x$ and $\lim \lambda_n=\lambda$, then $\{\lim \lambda_n x_n\}\ni \lambda x$.

§2. The neighbourhoods of zero. Let E be a linear ranked space. We will denote the system of neighbourhoods of 0 with rank n by $\mathfrak{V}_n$, and fundamental sequences with respect to 0 by $\{U_n\}$, $\{V_n\}$, $\cdots$. Obviously $\{\mathfrak{V}_n\}$ satisfies the axioms (A), (B), (a), (b) in [2].

Furthermore, from (I), (II), we get following properties.

(RL_1) For any $\{U_n\}$ and $\{V_n\}$, there is a $\{W_n\}$ such that $U_n+V_n\subseteq W_n$.

(RL_2) (i) For any $\{U_n\}$ and λ, there is a $\{V_n\}$ such that $\lambda U_n\subseteq V_n$.

(ii) For any x and $\{\lambda_n\}$ with $\lim \lambda_n=0$, there is a $\{V_n\}$ such that $\lambda_n x\in V_n$.

(iii) For any $\{U_n\}$ and $\{\lambda_n\}$ with $\lim \lambda_n=0$, there is a $\{V_n\}$ such that $\lambda_n U_n\subseteq V_n$.

(RL_3) Let x be any point in E. For any $\{U_n\}$ there is a $\{v_n(x)\}$ such that $x+U_n\subseteq v_n(x)$, and, conversely, for any $\{u_n(x)\}$ there is a $\{V_n\}$ such that $u_n(x)\subseteq x+V_n$.