52. *Realization of Irreducible Bounded Symmetric Domain of Type (V)*

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This note is a partial report of our researches concerning irreducible bounded symmetric domains of exceptional type. The detailed exposition with full proofs will be presented elsewhere.

1. It is well-known, since E. Cartan [1], that the irreducible bounded symmetric domain of type (V) is of 16-dimension and that of type (VI) is of 27-dimension; however no explicit descriptions of the *bounded* models of these domains have been clarified, as far as we know. In fact, several authors, M. Koecher, Pyateskii-Shapiro, U. Hirzebruch have tried to give the *unbounded* models of the domains using the theory of Jordan algebras and homogeneous cones. But, to determine the corresponding bounded models from the unbounded ones seems to be not so easy, though the general concept of Cayley transforms of bounded symmetric domains has been established by A. Koranyi and J. A. Wolf [6].

On the other hand, we have developed in [5] the *canonical* method of bounded realizations of general bounded symmetric domains as matrix-spaces; this yields, as special cases, the well-known bounded models of irreducible bounded symmetric domains of classical type (I)-(IV). We will apply it to the domain of type (V), and give the bounded model which is the *simplest one* in our sense.

2. Now we settle the notation that will be used. Let $\mathfrak{C}$ denote the algebra of Cayley numbers over the complex numbers C and $\mathfrak{J}$ the corresponding simple Jordan algebra of exceptional type [2], [8]; namely the complex vector space of all hermitian matrices of degree 3 over $\mathfrak{C}$ whose elements are written as follows:

$$(1) \qquad u=\begin{pmatrix}\xi_1, x_3, \bar{x}_2\\ \bar{x}_3, \xi_2, x_1\\ x_2, \bar{x}_1, \xi_3\end{pmatrix};\ x_i\in\mathfrak{C},\ \xi_i\in C.$$

($x\to\bar{x}$ denotes the involution in the sense of Cayley numbers). The Jordan product $u\cdot v$ in $\mathfrak{J}$ is, as usual, given by $u\cdot v=\frac{1}{2}(uv+vu)$, where uv is the ordinary matrix product. C. Chevalley and R. D. Schafer proved in [2] that the 27-dimensional irreducible representation $\mathfrak{G}$ of the complex simple Lie algebra $\mathfrak{g}_C$ of type E_6 are realized over the representation space $\mathfrak{J}$ as follows: