51. *On Some Homogeneous Boundary Value Problems Bounded Below*

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§1. Introduction. Let Ω be a compact oriented Riemannian n-space with smooth boundary Γ. Let A be a linear partial differential operator on Ω of order $2m$. We assume A is strongly elliptic, that is, there is a constant $C>0$ such that, for any x in Ω and for any non zero vector ξ cotangent to Ω at x, we have

$$C^{-1}|\xi|^{2m}\leq \operatorname{Re}\sigma_{2m}(A)(x,\xi)\leq C|\xi|^{2m},$$

where $\sigma_{2m}(A)$ is the principal symbol of A. We consider normal systems $\{B_r\}_{r\in R}$, $R=(r_1,r_2,\cdots,r_m)$, of m boundary operators B_{r_j}. r_j is the order of B_{r_j}. We assume $r_j<2m$ for any $j=0,1,\cdots,m$. The problem to be considered is

Problem 1. Characterize those couples $\{A,\{B_r\}_{r\in R}\}$ which give, with some constants $1/2\geq\varepsilon\geq 0$, $C,\beta>0$, the estimate

$$\operatorname{Re}((A+\beta)u,u)_{L^2(\Omega)}\geq C\|u\|^2_{H^{m-\varepsilon}(\Omega)} \tag{1}$$

for all u in $H^{2m}_B(\Omega)=\{u\in H^{2m}(\Omega)\,;\,B_ru|_\Gamma=0,\text{ for any } r\in R\}$.

Here $H^s(\Omega)$ denotes the Sobolev space on Ω of order s, $\|\ \ \|_{H^s(\Omega)}$ is its norm and $(\ \ ,\ \ )_{L^2(\Omega)}$ is the inner product in $L^2(\Omega)$.

If $1/2>\varepsilon\geq 0$, the problem was treated in far stronger form in [3]. In this note we concern with the case $\varepsilon=1/2$. So the problem is

Problem 1′. Characterize those couples $\{A,\{B_r\}_{r\in R}\}$ which give, with some constants $C,\beta>0$, the estimate

$$\operatorname{Re}((A+\beta)u,u)_{L^2(\Omega)}\geq C\|u\|^2_{H^{m-1/2}(\Omega)} \tag{2}$$

for all u in $H^{2m}_B(\Omega)$.

We assume the following hypothesis (H) that was proved in the case $0\leq\varepsilon<1/2$ necessary for the estimate (1) to hold. (See [3] and [6].)

(H) The set R coincides with one of the R_j's defined by $R_j=(0,1,\cdots\cdots,m-j-1,m,m+1,\cdots,m+j-1)$, $1\leq j\leq m$. Under this hypothesis we give a necessary and sufficient condition for the estimate (2) to hold.

Proofs are omitted. Detailed discussions will be published elsewhere.*)

§2. Results. We denote by ν the interior unit normal to Γ and

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