50. *Some Properties of Regular Distribution Semi-groups**)

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The notion of distribution semi-groups was defined by J. L. Lions in [4]. D. Fujiwara characterized the infinitesimal generator of an exponential distribution semi-group in terms of an equi-continuous semi-group in some Fréchet space in [3]. In this paper we shall report that Fujiwara's results can be partially extended to the infinitesimal generator of a regular distribution semi-group if we introduce the notion of a locally equi-continuous semi-group, which was recently studied by T. Kōmura in [3]. In order to characterize the infinitesimal generator of a locally equi-continuous semi-group in a locally convex space, she used the concept of the generalized Laplace transform of a distribution. This concept is also essential to the present work. Complete proofs of the theorems in the present note will be published elsewhere.

§1. Statements of the results. Let $L(E, F)$ be the totality of continuous linear mappings from E to F, where E and F are topological vector spaces. The set $L(E, E)$ is denoted by $L(E)$. Let us abbreviate Schwartz space $\mathcal{D}(R^1)$ by $\mathcal{D}$. Let X be a Banach space. Consider the totality of X-valued distributions, $\mathcal{D}'(X)=L(\mathcal{D}, X)$. Let $\mathcal{D}_+$(or $\mathcal{D}'_+(X)$) be the totality of elements of $\mathcal{D}$(or $\mathcal{D}'(X)$) whose supports are contained in $[0, \infty)$. For any linear operator T, we denote its domain (or range, or null space) by $D(T)$ (or $R(T)$, or $N(T)$).

Following Lions, we say that an $L(X)$-valued distribution $\mathcal{T}$ is a regular distribution semi-group (D.S.G., in short) if $\mathcal{T}$ satisfies the following five conditions.

(T.1) $\mathcal{T} \in \mathcal{D}'_+(L(X))$.

(T.2) $\mathcal{T}(\phi*\psi)=\mathcal{T}(\phi)\mathcal{T}(\psi)$ *if* $\phi, \psi \in \mathcal{D}_+$.

(T.3) $\bigcap_{\phi \in \mathcal{D}_+} N(\mathcal{T}(\phi))=\{0\}$.

(T.4) *The linear hull* $\mathcal{R}$ *of* $\bigcup_{\phi \in \mathcal{D}_+} R(\mathcal{T}(\phi))$ *is dense in* X.

(T.5) *For any* $x \in \mathcal{R}$, *there exists an* X-*valued function* $x(t)$ *such that*: (i) $x(t)=0$ *for* $t<0$, (ii) $x(0)=x$, (iii) $x(t)$ *is continuous for* $t\geqq 0$, (iv) $\mathcal{T}(\phi)x=\int_0^\infty \phi(t)x(t)dt$ *for any* $\phi \in \mathcal{D}$.

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