144. *A Class of Markov Processes with Interactions. I*[1]

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Here, we consider a motion of one particle under the interactions between an infinite number of similar particles. Each particle moves independently in a Markovian way until an exponential jumping time comes, and it jumps with a hitting measure which depends on other particles. A model, where the jumping time also depends on other particles, is discussed under auxiliary conditions. These results extend [9].

The models here came into our interest through the works of McKean [3–5], which started with Kac's model of Boltzmann equation [2].

1. Let $P(s, x, t, E)$ be a transition probability on a locally compact space R with countable bases and topological Borel field $\boldsymbol{B}(R)$. Assume $P(s, x, t, R) \equiv 1$ and

(1) $\qquad P(s, x, t, U) \to 1, \quad$ as $t-s \to 0$, for open U containing x.

Let $q(t, y)$ be a non-negative, measurable function, bounded on compact (t, y)-sets. Define

$$(2) \qquad P_0(s, x, t, E) = E_{s,x}\left(\exp\left[-\int_s^t q(\sigma, X_\sigma(w))d\sigma\right]\chi_E(X_t(w))\right),$$

where $X_t(w)$ is a measurable Markov process with transition probability $P(s, x, t, E)$. $E_{s,x}(\cdot)$ is the expectation conditioned that the particle starts at x at time s. This set up is possible by (1). Let $q_n(t, y)$, $n=0, 1, \cdots$ be non-negative, measurable and $q(t, y) \equiv \sum_{n=0}^{\infty} q_n(t, y)$, and let $\pi_n(y_1, \cdots, y_n \mid t, y)$ be probability measures on $(R, \boldsymbol{B}(R))$, measurable in $(y_1, \cdots, y_n, t, y)$ for fixed $E \in \boldsymbol{B}(R)$.[2]

Consider a *forward equation* and a version of *backward equation*:

$$(3) \quad P^{(f)}(s, x, t, E) = P_0(s, x, t, E) + \int_s^t d\tau \int_R P^{(f)}(s, x, \tau, dy) \sum_{n=0}^{\infty} q_n(t, y) \int_{R^n} \prod_{k=1}^{n} P_{s,\tau}^{(f)}(dy_k) \times \int_R \pi_n(y_1, \cdots, y_n \mid \tau, y, dz)\, P_0(\tau, z, t, E),^{3)}$$

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2) For the intuitive meanings of the quantities, the reader can consult [9].

3) The 0-th term of the sum is $q_0(\tau, y) \int_R \pi_0(\tau, y, dz)\, P_0(\tau, z, t, E)$.