190. *Distributions as the Boundary Values of Analytic Functions*

By Richard D. CARMICHAEL
Department of Mathematics, Virginia Polytechnic Institute, Blacksburg, Virginia, U. S. A.

(Comm. by Kinjirô KUNUGI, M. J. A., Dec. 12, 1969)

The aim of this note is to announce some theorems (Theorems 1–8) concerning distributions as the boundary values of functions which are analytic in a subset of $\boldsymbol{C}^n$.

The n dimensional notation used here will be that of Schwartz [1]. $C \subseteq \boldsymbol{R}^n$ is a cone with vertex at zero if $y \in C$ implies $\lambda y \in C$ for all positive scalars λ. The intersection of C with the unit sphere $|y|=1$ is called the projection of C and is denoted $\mathrm{pr}\, C$. Let C' be a cone such that $\mathrm{pr}\, \bar{C}' \subset \mathrm{pr}\, C$; then C' will be called a compact subcone of C. The function

$$u_C(t) = \sup_{y \in \mathrm{pr} C} (-\langle t, y\rangle)$$

is the indicatrix of the cone C. $0(C)$ will denote the convex envelope of C. $T^C = \boldsymbol{R}^n + iC$, where C is an open connected cone, is a tubular radial domain. The Fourier transform of $f(t) \in L^1$ will be denoted by $\hat{f}$ or $\mathscr{F}[f(t); x]$ and is defined as

$$\hat{f}(x) = \int_{\boldsymbol{R}^n} f(t) e^{2\pi i \langle x, t\rangle} dt.$$

We refer to Schwartz [1] and Gel'fand and Shilov [2] for definitions and facts concerning the distribution spaces.

1. Distribution boundary values in Z'. Lauwerier [3] has shown that functions which are analytic in $\mathrm{Im}\,(z) > 0$, $z \in \boldsymbol{C}^1$, and which are bounded by a polynomial have distributional boundary values in the Z' topology. We extend the results of Lauwerier to functions which are analytic in tubular radial domains, T^C.

Theorem 1. *Let $f(z)$ be analytic in T^C. For any arbitrary compact subcone C' of C let $f(z)$ satisfy*

$$|f(z)| \leq K(C')(1+|z|)^N e^{2\pi(A+\sigma)|y|}, \; z \in T^{C'}, \tag{1}$$

for all $\sigma > 0$, where A is a nonnegative real number, N is any real number, and $K(C')$ is a constant depending on C'. Then $f(z)$ has a distributional boundary value $U \in Z'$ which is the Fourier transform of an element $V \in \mathscr{D}'$ which vanishes for $u_C(t) > A$.

Let P be a constant such that $N - 2P \leq -n - \varepsilon$ for all $\varepsilon > 0$; and let $B \in \boldsymbol{R}^1$, $B > 0$, be such that