186. *Realization of Irreducible Bounded Symmetric Domain of Type* (VI)

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1. This is a continuation of our preceding note [3] which appeared in these Proceedings. We shall present here, without proof, the *canonical bounded model* of the irreducible bounded symmetric domain of exceptional type (VI) in the sense of [4].

As was pointed out in [4], we need at first to describe explicitly the irreducible representation of the complex simple Lie algebra of type E_7 which is of the lowest degree, 56. Such a representation was previously discussed by several authors, for instance by H. Freudenthal; however a presentation of that representation which suited our purpose was recently given by R. B. Brown [1] for the first time. His result will be, therefore, briefly reproduced in the following sections 2-3. As for the notation we refer the reader to [3], [4].

2. Let $\mathfrak{J}$ denote the exceptional simple Jordan algebra as described in [1]-[3]; namely $\mathfrak{J}$ is the totality of the (3.3)-hermitian matrices over the complex Cayley numbers $\mathfrak{C}$. The canonical non-degenerate inner-product (u, v) in $\mathfrak{J}$ will be introduced by $(u, v)=\mathrm{Trace}$ $(u\circ v)$, $(u, v\in\mathfrak{J})$ (cf. [1], [2], [5]), for which we consider the dual $\mathfrak{J}^*$ of $\mathfrak{J}$ and will identify hereafter $\mathfrak{J}^*$ with $\mathfrak{J}$ through this inner-product. Now we introduce a 56-dimensional complex vector space V by putting

$$V=V_1\oplus V_2\oplus V_3\oplus V_4, \tag{1}$$

where both V_1 and V_4 are of 1-dimension and $V_2=\mathfrak{J}^*$, $V_3=\mathfrak{J}$. The element x of V is then written as

$$x=\alpha f_1+u^*+v+\beta f_2;\ \alpha, \beta\in C,\ u, v\in\mathfrak{J}, \tag{2}$$

where f_1, f_2 denote, respectively, the generators of V_1, V_4 and $u^*\in\mathfrak{J}^*$ is defined by $u^*(v)=(u, v)$ for all $v\in\mathfrak{J}$. After R. B. Brown we introduce in V a non-associative algebra structure $\mathfrak{B}$ by the following rule:

i) $f_if_i=f_1$ $(i=1, 2)$, $\quad f_1f_2=f_2f_1=0$

ii) $f_1u=\frac{1}{3}u$, $\quad f_2u=\frac{2}{3}$; $\quad f_1v^*=\frac{2}{3}v^*$, $\quad f_2v^*=\frac{1}{3}v^*$

iii) $uf_1=0$, $\quad uf_2=u$; $\quad v^*f_1=v^*$, $\quad v^*f_2=0$

iv) $uv^*=(u, v)f_1$, $\quad u^*v=(u, v)f_2$

v) $uv=2(u\times v)^*$, $\quad u^*v^*=2(u\times v)$

$(u, v\in\mathfrak{J})$, where the crossed product $u\times v$ in $\mathfrak{J}$ is given through $(u\times v, w)=3(u, v, w)$ (for $w\in\mathfrak{J}$), the right hand side being the tri-linear form on $\mathfrak{J}$ obtained by linearizing the cubic from on $\mathfrak{J}$ (see, [1], [5]):