10. *Characterizations of Strongly Regular Rings*

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In this note by a ring we shall mean a not necessarily commutative but associative ring and by radical of the ring we mean the Jacobson radical (see N. Jacobson [5]). Following J. von Neumann [13] we shall say that the ring A is regular if, for every element a of A, there exists an element x in A such that $a=axa$. It is well known that the class of regular rings plays a very important role in the abstract algebra, in the theory of Banach algebras (cf. C. E. Rickart [15]) and in the continuous geometry (see J. von Neumann [14]). An interesting result is that the ring of all linear transformations of a vector space over a division ring is a regular ring. Some ideal-theoretical characterizations of regular rings have been obtained by L. Kovács [7] and J. Luh [11].

A ring A is called strongly regular if to every element a of A there exists at least one element x in A such that $a=a^2x$ (See R. F. Arens and I. Kaplansky [2]). It can be seen that every strongly regular is regular (see T. Kandô [6]). Following E. Hille [4] a ring A is said to be a two-sided ring if every one-sided (left or right) ideal of A is a two-sided ideal of A. Evidently every division ring and every commutative ring is a two-sided ring. It is easy to see that there exists two-sided ring which is neither commutative nor a division ring. Two-sided rings called as duo rings have formerly been investigated by E. H. Feller [3], G. Thierrin [17] and S. Lajos [8]. Thierrin using the classical method of N. H. McCoy [12] has verified that every two-sided ring can be represented as a subdirect sum of subdirectly irreducible two-sided rings.

First named author has recently obtained some ideal-theoretical characterizations of two-sided regular rings which are analogous to his characterizations of semilattices of groups (see S. Lajos [8]–[10]). S. Lajos' earlier criteria are contained in the following result which will be stated here with no proof.

Theorem. *For an associative ring A the following eleven conditions are equivalent with each other:*

(I) *A is a strongly regular ring.*

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