3. *On wM-Spaces.* I

By Tadashi ISHII
Utsunomiya University

(Comm. by Kinjirô KUNUGI, M. J. A., Jan. 12, 1970)

1. Introduction. The purpose of the present paper is to introduce the notion of wM-spaces, which is a generalization of M-spaces introduced by K. Morita [6], and to show some preperties of these spaces. For a sequence $\{\mathfrak{A}_n\}$ of open (or closed) coverings of a topological space X, we shall consider the following two conditions:

(M_1) $\begin{cases} \text{If } \{K_n\} \text{ is a decreasing sequence of non-empty subsets of } X \text{ such} \\ \text{that } K_n \subset \mathrm{St}(x_0, \mathfrak{A}_n) \text{ for each } n \text{ and for some point } x_0 \text{ of } X\text{, then} \\ \cap \bar{K}_n \neq \emptyset. \end{cases}$

(M_2) $\begin{cases} \text{If } \{K_n\} \text{ is a decreasing sequence of non-empty subsets of } X \text{ such} \\ \text{that } K_n \subset \mathrm{St}^2(x_0, \mathfrak{A}_n) \text{ for each } n \text{ and for some point } x_0 \text{ of } X\text{, then} \\ \cap \bar{K} \neq \emptyset.\text{[1)]} \end{cases}$

A space X is an M-space if there exists a normal sequence $\{\mathfrak{A}_n\}$ of open coverings of X satisfying (M_1). A space X is an M^*-space ($M^\sharp$-space) if there exists a sequence $\{\mathfrak{F}_n\}$ of locally finite (closure preserving) closed coverings of X satisfying (M_1) (T. Ishii [2], F. Siwiec and J. Nagata [8]). A space X is a $w\Delta$-space if there exists a sequence $\{\mathfrak{A}_n\}$ of open coverings of X satisfying (M_1) (C. Borges [1]). As is shown by K. Morita [7], there exists an M^*-space which is locally compact Hausdorff but not an M-space. Further, in our previous paper [3], we proved that a normal space X is an M-space if and only if it is an M^*-space.

Now we shall define wM-spaces including all M-spaces, M^*-spaces and $M^\sharp$-spaces.

Definition. A space X is a wM-space if there exists a sequence $\{\mathfrak{A}_n\}$ of open coverings of X satisfying (M_2).

In the above definition, we may assume without loss of generality that $\mathfrak{A}_{n+1}$ refines $\mathfrak{A}_n$ for each n.

As a remarkable property of a wM-space, we can prove that every normal wM-space is strongly normal, that is, collectionwise normal and countably paracompact (Theorem 2.4). This result plays an important role in metrizability of wM-spaces in the next paper. Throughout this paper we assume at least T_1 for every topological spaces unless otherwise specified.

1) For each positive integer k, $\mathrm{St}^k(x_0, \mathfrak{A}_n)$ denotes the iterated star of a point x_0 in each covering $\mathfrak{A}_n$.