1. *On Functions of Yosida's Class* (A)

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1. Let $f(z)$ be a non-rational meromorphic function in $|z|<\infty$ and $\rho(f(z))$ the spherical derivative of $f(z)$. Following K. Yosida [2], we say that $f(z)$ belongs to the class (A) if for any sequence of complex numbers $\{a_n\}$, the family of functions

$$\{f(z+a_n)\}, \qquad n=1,2,\cdots, \tag{1}$$

is normal in the sense of Montel $|z|<\infty$. If, in addition, any family of the form (1) admits no constant limit, we say that $f(z)$ belongs to the subclass (A_0) [the functions of the 1st category in Yosida's terminology]. The subclass (A_0) contains, in particular, an important class of meromorphic functions as the doubly periodic functions.

Yosida [2] has proved that $f(z)$ belongs to (A) if and only if

$$\rho(f(z))=O(1), \qquad z\to\infty.$$

Among the other results, he has proved that a function of the subclass (A_0) possesses no Nevanlinna deficient value. In [1] the author has pointed out that Yosida's results allow to prove that a function of (A_0) admits no Valiron deficient value. The present note contains the details.

2. Using the standard terminology of the Nevanlinna theory, the deficiency of Valiron $\delta(a,f)$ of a value a is defined as follows:

$$\delta(a,f)=\overline{\lim_{r\to\infty}}\frac{m(r,a,f)}{T(r,f)}.$$

If $\delta(a,f)>0$, the value a is said to be a Valiron deficient value for $f(z)$.

Theorem. *If $f(z)$ belongs to (A_0), then $\delta(a,f)=0$ for any complex a (finite or infinite).*

Proof. Yosida [2] has proved that for a function $f(z)\in(A_0)$ and for a set of complex values $a_1,a_2,\cdots,a_q$ $(q\geq 3)$,

$$\sum_{i=1}^{q} m(r,a_i,f)=O(r)+S(r)$$

holds with $S(r)=o(T(r,f))$. Our theorem will be proved if we show that

$$\lim_{r\to\infty}\frac{T(r,f)}{r^2}>0 \tag{2}$$

is valid for any $f(z)\in(A_0)$.

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