27. *Axioms for Commutative Rings*

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G. R. Blakley, S. Ôhashi and K. Iséki gave some new definitions of commutative rings and semirings (see [1]-[3]). In this note, we shall give other difinitions of commutative rings with unity and semirings with zero and unity, where two binary operations are commutative.

Theorem 1. *A set with two nullary operations,* 0 *and* 1, *with one unary operation,* $-$, *and with two binary operations,* $+$ *and juxtaposition, such that*

1.1) $r+0=r$,

1.2) $r1=r$,

1.3) $((-r)+r)a=0$,

1.4) $(a+(b+cz))r=(br+ar)+z(cr)$

for any a, b, c, r, z, *is a commutative ring with unity.*

Proof. We can prove this theorem as follows.

1.5) $(-r)+r=0$ (See [1])

1.6) $0a=0$ (See [1])

1.7) $a+b$

$=(a+(b+00))1$ by 1.6, 1.1, 1.2.

$=(b1+a1)+0(01)$ by 1.4.

$=b+a$ by 1.2, 1.6, 1.1.

1.8) cz

$=(0+(0+cz))1$ by 1.7, 1.1, 1.2.

$=(01+01)+z(c1)$ by 1.4.

$=zc$ by 1.7, 1.2, 1.1.

1.9) $a+(b+c)$

$=(a+(b+c1))1$ by 1.2.

$=(b1+a1)+1(c1)$ by 1.4.

$=(a+b)+c$ by 1.7, 1.8, 1.2.

1.10) $(zc)r$

$=(0+(0+cz))r$ by 1.8, 1.7, 1.1.

$=(0r+0r)+z(cr)$ by 1.4.

$=z(cr)$ by 1.6, 1.7, 1.1.

1.11) $(b+c)r$

$=(0+(b+c1))r$ by 1.7, 1.1, 1.2.

$=(br+0r)+1(cr)$ by 1.4.

$=br+cr$ by 1.6, 1.1, 1.8, 1.2.