26. *On Definition for Commutative Idempotent Semirings*

By Sakiko ÔHASHI

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Recently Professor S. Tamura (2) gave some new axioms for commutative rings and semirings. In this Note, we shall give some axiom systems for commutative idempotent semirings. By idempotent semirings, we mean that the addition and the multiplication are both idempotent. This class of semirings is very important in algebraic systems.

First of all, I give a remark. In my previous paper (1), the terminology "distributive lattice" in Theorem 2 and its proof should be replaced by "commutative idempotent semiring".

Let $\langle X, +, \cdot, 0, 1\rangle$ be an algebraic system, where 0 and 1 are elements of X, $+$ and $\cdot$ are binary operations on X. As in the previous paper (1), we denote $a \cdot b$ by ab.

Theorem 1. $\langle X, +, \cdot, 0, 1\rangle$ *is a commutative idempotent semiring, if and only if it satisfies the following conditions:*

1.1) $r+0=r$,

1.2) $r1=r$,

1.3) $0a=0$,

1.4) $((a+br)^{+}cz+d+d)r=br+(ar+z(cr)+dr)$

for every a, b, c, d, r, z.

Proof. It is obvious that every commutative idempotent semiring satisfies 1.1)–1.4). We shall prove the "if" part.

1.5) $a+b=((a+b1)+00+0+0)1$ $\{2, 3, 1\}$
$=b1+(a1+0(01)+01)$ $\{4\}$
$=b+a.$ $\{2, 3, 1\}$

1.6) $cz=((0+01)+cz+0+0)1$ $\{1, 5, 3, 2\}$
$=01+(01+z(c1)+01)$ $\{4\}$
$=zc.$ $\{3, 1, 5, 2\}$

1.7) $(b+a)+c=(a+b)+c$ $\{5\}$
$=((a+b1)+c1+0+0)1$ $\{2, 1\}$
$=b1+(a1+1(c1)+01)$ $\{4\}$
$=b+(a+c).$ $\{2, 6, 3, 1\}$

1.8) $(cz)r=((0+0r)+zc+0+0)r$ $\{6, 1, 3, 5\}$
$=0r+(0r+c(zr)+0r)$ $\{4\}$
$=c(zr).$ $\{3, 1, 5\}$

1.9) $(a+c)r=((a+0r)+c1+0+0)r$ $\{3, 2, 1, 5\}$