48. *Abelian Groups and $\mathfrak{N}$-Semigroups*[*)]

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§1. Introduction. A commutative archimedean cancellative semigroup without idempotent is called an $\mathfrak{N}$-semigroup. The author obtained the following [3 or 1, p. 136].

Theorem 1. *Let K be an abelian group and N be the set of all non-negative integers. Let I be a function $K\times K\rightarrow N$ which satisfies the following conditions:*

(1) $I(\alpha,\beta)=I(\beta,\alpha)$ *for all* $\alpha,\beta\in K$.

(2) $I(\alpha,\beta)+I(\alpha\beta,\gamma)=I(\alpha,\beta\gamma)+I(\beta,\gamma)$ *for all* $\alpha,\beta,\gamma\in K$.

(3) $I(\varepsilon,\varepsilon)=1$. *$\varepsilon$ being the identity element of K.*

(4) *For every $\alpha\in K$ there is a positive integer m such that* $I(\alpha^m,\alpha)>0$.

We define an operation on the set $S=N\times K=\{(m,\alpha)\colon m\in N,\alpha\in K\}$ by $(m,\alpha)(n,\beta)=(m+n+I(\alpha,\beta),\alpha\beta)$.

Then S is an $\mathfrak{N}$-semigroup. Every $\mathfrak{N}$-semigroup is obtained in this manner. S is denoted by $S=(K;I)$.

To prove the theorem in [3] we used the fact $\bigcap_{n=1}^{\infty}a^nS=\emptyset$, and a group-congruence was defined, which are still effective in the case where cancellation is not assumed. However the quotient group of an $\mathfrak{N}$-semigroup gives another proof of the latter half of the theorem. In this paper we study the relationship between an $\mathfrak{N}$-semigroup and abelian group as the quotient group. Theorm 2 states the relationship between the I-function of an $\mathfrak{N}$-semigroup and the factor system of the quotient group as the extension. Theorem 4 is the main theorem of this paper, which asserts the existence of maximal $\mathfrak{N}$-subsemigroups of a given abelian group. Theorem 5 is an application of Theorem 4 to the extension theory of abelian groups.

§2. Proof of a part of the latter half of Theorem 1. Let S be an $\mathfrak{N}$-semigroup and G be the quotient group of S, i.e. the smallest group into which S can be embedded. We may assume $S\subset G$. Let $a\in S$. The element a is of infinite order in G. Let A be the infinite cyclic group generated by a. Let $G_a=G/A$. G_a is called the structure group of S with respect to a. G is the disjoint union of the congruence classes of G modulo A.

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