121. *Paracompactifications of M-spaces*

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By a space we shall always mean a completely regular Hausdorff space unless otherwise specified.

1. Let X be a space with a uniformity Φ agreeing with the topology of X; that is, Φ is a family of open coverings of X satisfying conditions (a) to (c) below, where for coverings $\mathfrak{U}$ and $\mathfrak{V}$ of X we mean by $\mathfrak{U}<\mathfrak{V}$ that $\mathfrak{V}$ is a refinement of $\mathfrak{U}$.

(a) If $\mathfrak{U}, \mathfrak{V} \in \Phi$, then there exists $\mathfrak{W} \in \Phi$ such that $\mathfrak{U}<\mathfrak{W}$ and $\mathfrak{V}<\mathfrak{W}$.

(b) If $\mathfrak{U} \in \Phi$, there is $\mathfrak{V} \in \Phi$ which is a star-refinement of $\mathfrak{U}$.

(c) $\{\mathrm{St}(x, \mathfrak{U}) \mid \mathfrak{U} \in \Phi\}$ is a basis of neighborhoods at each point x of X.

Let $\{\Phi_\lambda \mid \lambda \in \Lambda\}$ be the totality of those normal sequences which consist of open coverings of X contained in Φ. Let $\Phi_\lambda=\{\mathfrak{U}_{\lambda i} \mid i=1,2,\cdots\}$, where $\mathfrak{U}_{\lambda i}$ is a star-refinement of $\mathfrak{U}_{\lambda, i-1}$ for $i=2,3,\cdots$. As in [1], we denote by (X, Φ_λ) the topological space obtained from X by taking $\{\mathrm{St}(x, \mathfrak{U}_{\lambda i}) \mid i=1,2,\cdots\}$ as a basis of neighborhoods at each point x of X. Let X/Φ_λ be the quotient space obtained from (X, Φ_λ) by defining those two points x and y equivalent for which $y \in \mathrm{St}(x, \mathfrak{U}_{\lambda i})$, for $i=1,2,\cdots$. Then there is a canonical map $\varphi_\lambda: X \to X/\Phi_\lambda$ which is continuous, and X/Φ_λ is metrizable.

Now we shall define a partial order in $\{\Phi_\lambda \mid \lambda \in \Lambda\}$. If each member of Φ_λ has a refinement in Φ_μ, we write $\Phi_\lambda<\Phi_\mu$. Then, if $\Phi_\lambda<\Phi_\mu$, there exists a continuous map $\varphi_\lambda^\mu: X/\Phi_\mu \to X/\Phi_\lambda$ such that $\varphi_\lambda=\varphi_\lambda^\mu \circ \varphi_\mu$, and $\{X/\Phi_\lambda; \varphi_\lambda^\mu\}$ is an inverse system of metrizable spaces. Let us set

$$\mu_\Phi(X)=\varprojlim X/\Phi_\lambda.$$

For any point x of X, let us put $\varphi(x)=\{\varphi_\lambda(x)\}$. Then $\varphi: X \to \mu_\Phi(X)$ is a homeomorphism into.

In case every Cauchy family $\{C_\gamma\}$ of X with respect to Φ which has the countable intersection property is non-vanishing (that is, $\cap \bar{C}_\gamma \neq \phi$), we say that X is *weakly complete* with respect to Φ.

Theorem 1. *The map $\varphi: X \to \mu_\Phi(X)$ is onto if and only if X is weakly complete with respect to Φ.*

In case Φ is the finest uniformity (that is, Φ consists of all normal open coverings of X), we write $\mu(X)$ instead of $\mu_\Phi(X)$. In this case