119. *On the Bi-ideals in Associative Rings*

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By a ring we shall mean an arbitrary associative ring. For the terminology not defined here we refer to N. Jacobson [3] and N. H. McCoy [8]. We announce some properties of bi-ideals in rings which are analogous to some properties of bi-ideals in semigroups.

For the subsets X and Y of a ring A by the product XY we mean the subring of A which is generated by the set of all products xy, where $x \in X$, $y \in Y$. By a bi-ideal B of A we mean a subring B of A satisfying the condition

(1) $$BAB \subseteq B.$$

Obviously every one-sided ideal of A is a bi-ideal and the intersection of a left and a right ideal of A is also a bi-ideal. It may be remarked that the notion of the bi-ideal in semigroups is a special case of the (m, n)-ideal introduced by S. Lajos [4]. The notion of bi-ideal for associative rings was earlier mentioned by S. Lajos [5]. He noted that the set of all bi-ideals of a regular ring is a multiplicative semigroup. The concept of the bi-ideal was introduced by R. A. Good and D. R. Hughes [1]. An interesting particular case of the bi-ideal is the notion of quasi-ideal due to O. Steinfeld [9] which is defined as follows. A submodule Q of a ring A is called a quasi-ideal of A if the following condition holds:

(2) $$AQ \cap QA \subseteq Q.$$

It is known that the product of any two quasi-ideals is a bi-ideal (see S. Lajos [5]). It may be remarked that in case of regular rings the notions of bi-ideal and quasi-ideal coincide.

In the following we formulate some general properties of bi-ideals in rings and characterize two important classes of rings in terms of bi-ideals.

Proposition 1. *The intersection of an arbitrary set of bi-ideals B_i $(i \in I)$ of a ring A is again a bi-ideal of A.*

Proposition 2. *The intersection of a bi-ideal B of a ring A and a subring S of A is a bi-ideal of the ring S.*

Proposition 3. *For an arbitrary subset T of a ring A and for a bi-ideal B of A the products BT and TB are bi-ideals of A.*

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