117. On the Spaces with the σ-Star Finite Open Basis

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(Comm. by Kinjirô KUNUGI, M. J. A., June 12, 1970)

One of the well known theorems for the metrizability is as follows: A regular T_1-space X is metrizable if and only if there exists a σ-locally finite open basis of X.

Our purpose of this paper is to study the spaces with the σ-star finite open basis.

Let us recall the definitions of terms which are used in the statement of this paper. Let X be a topological space and $\mathfrak{A}$ be a collection of subsets of X. $\mathfrak{A}$ is said to be *point finite* (resp. *point countable*) if every point of X is contained in at most finitely (resp. countably) many elements of $\mathfrak{A}$. $\mathfrak{A}$ is *locally finite* (resp. *locally countable*) if every point of X has a neighborhood which intersects only finitely (resp. countably) many elements of $\mathfrak{A}$. $\mathfrak{A}$ is *star finite* (resp. *star countable*) if every element of $\mathfrak{A}$ intersects only finitely (resp. countably) many elements of $\mathfrak{A}$. A space X is said to be *strongly paracompact* if every open covering of X has a star finite open covering of X as a refinement. *A σ-star finite open basis* is an open basis which is the union of countably many star finite open coverings.

Finally, to state our results we need the next notation. Let $\{U_x \mid x \in X\}$ be a collection of subsets of X with the index set X, then its collection is symmetric if "$y \in U_x$" is equivalent to "$x \in U_y$".

We assume that all the spaces in this paper are T_1-spaces and for a symmetric collection $\{U_x \mid x \in X\}$, U_x contains x for every point $x \in X$.

As is well known, not every metric space has a σ-star finite basis (see Yu. M. Smirnov [5]). The existence of a σ-star finite open basis is not sufficient for a metric space to be strongly paracompact (see J. Nagata [4, p. 201]), but clearly, a strong paracompactness or a local compactness is sufficient for a metric space to be with the σ-star finite open basis, and furthermore it is known that a metric space X has a σ-star finite open basis if and only if X is homeomorphic to a subspace of a topological product $N(\Omega)$[1] $\times I^{w}$ for suitable Ω (see J. Nagata [4, p. 201] or [3]).

1) $N(\Omega)$ is the *generalized Baire's zero dimensional space* with respect to Ω, that is $N(\Omega)$ is the set of all sequences $(\alpha_1, \alpha_2, \cdots)$ of elements $\alpha_i \in \Omega$. The distance between two distinct points $\alpha=(\alpha_1, \alpha_2, \cdots)$ and $\beta=(\beta_1, \beta_2 \cdots)$ of $N(\Omega)$ are defined by
$$\rho(\alpha, \beta)=\frac{1}{\min\{k \mid \alpha_k \neq \beta_k\}}.$$