150. *Absolute Nörlund Summability Factor of Fourier Series*

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1. Let $\sum_{n=0}^{\infty} a_n$ be an infinite series and let (s_n) be the sequence of its partial sums. Let $(p_n)=(p_0, p_1, \cdots)$ be a sequence of positive numbers and $P_n=p_0+p_1+\cdots+p_n$ $(n=0,1,2,\cdots)$, $p_{-1}=P_{-1}=0$. We write

$$t_n=P_n^{-1}\sum_{k=0}^{n} p_{n-k}s_k=P_n^{-1}\sum_{k=0}^{n} P_{n-k}a_k \qquad (n=1,2,\cdots)$$

which is called the nth Nörlund mean of the series $\sum a_n$ or the sequence (s_n). If the sequence (t_n) is of bounded variation, then the series $\sum a_n$ is called to be absolutely summable (N, p_n) or summable $|N, p_n|$ and we write $\sum a_n \in |N, p_n|$.

Let f be an integrable function over the interval $(0, 2\pi)$ and be periodic with period 2π. We denote its Fourier series by

$$f(t)\sim \frac{1}{2}a_0+\sum_{n=1}^{\infty}(a_n \cos nt+b_n \sin nt)=\sum_{n=0}^{\infty} A_n(t).$$

The sequence (m_n) is called the absolute Nörlund summability factor or the $|N, p_n|$ summability factor of the Fourier series of f at the point x if $\sum m_n A_n(x) \in |N, p_n|$.

We suppose always that all m_n are non-negative.

S. V. Kolhekar [1] has proved the

Theorem A. *Let (m_n) be a monotone decreasing sequence satisfying the condition*

$$(1) \qquad \sum_{n=1}^{\infty} m_n n^{-1} \log n < \infty$$

and let (p_n) be a monotone increasing sequence such that

$$(2) \qquad p_n/P_n=O(1/n), \qquad \Delta(P_n/p_n)=O(1) \qquad \text{as } n\to\infty.$$

Then, if

$$(3) \qquad \Phi(t)=\int_0^t |\varphi(u)|\,du=O(t) \qquad \text{as } t\to 0$$

where $\varphi(u)=\varphi_x(u)=f(x+u)+f(x-u)-2f(x)$, *then* $\sum m_n A_n(x) \in |N, p_n|$.

We define a function $m(t)$ continuous on the interval $(1,\infty)$ such that $m(n)=m_n$ for $n=1,2,\cdots$ and $m(t)$ is linear for every non-integral t. Similarly $p(t)$ is defined by the sequence (p_n) and we put $P(t)=\int_0^t p(u)du$.