191. *Noetherian QF-3 Rings and Two-sided Quasi-Frobenius Maximal Quotient Rings**)

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The notion of QF-3 rings was introduced originally by R. M. Thrall [13] for the case of finite dimensional algebras over a field. Recently this notion has been extended to the case of general rings in various ways (cf. [2], [5], [12]). For example, a ring A is called left QF-3 or left QF-3′ according as A has a faithful, projective injective left ideal or the injective envelope of the left A-module $A, E({}_AA)$, is torsionless, and a ring which is left QF-3 by any one of the definitions given in the literature is left QF-3′. The notion of QF-3′ rings, however, does not seem to fit Noetherian rings.

In this paper we shall call a ring A a left QF-3 ring if $E({}_AA)$ is flat. Right QF-3 rings are defined similarly. For example, the ring of rational integers is left QF-3 in our sense, but not left QF-3′ in the sense mentioned above. As for Noetherian QF-3 rings, we shall prove the following theorems.

Theorem 1. *Let A be a left Noetherian ring. If A is left QF-3, then A is right QF-3.*

Theorem 2. *Let A be a left Noetherian, left QF-3 ring. Then we have*

$$(1)\qquad \mathrm{Hom}_A([\mathrm{Ext}_A^n({}_AX, {}_AA)]_A, E(A_A))=0, \qquad n=1,2,\cdots$$

for every finitely generated left A-module X.

According to Jans [4], the dual of every finitely generated right A-module is reflexive if and only if

$$(2)\qquad \mathrm{Hom}_A([\mathrm{Ext}_A^1(X, {}_AA)]_A, A_A)=0$$

for every finitely generated torsionless left A-module X.[1] Hence we obtain from Theorem 2 the following

Corollary 3. *Let A be a left Noetherian left QF-3 ring. Then the dual of every finitely generated right A-module is reflexive.*

The above notion of QF-3 rings is useful also for non-Noetherian rings.

As is known as a theorem of R. E. Johnson, the maximal left

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*) Dedicated to Professor K. Asano on his sixtieth birthday.

1) This result is true without any finiteness condition on A, although Jans assumes A to be left and right Noetherian. This fact has already been used in our previous paper [9].