208. *Construction of Elementary Solutions for I-hyperbolic Operators and Solutions with Small Singularities*

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In this note we treat the following problems:

(I) Construction of the elementary solution of the Cauchy problem for a hyperbolic differential operator (Theorem 3).

(II) The condition for I-hyperbolicity (Theorem 5).

(III) Construction of a solution of a (homogeneous) differential equation whose singular support on S^*M is contained in a bicharacteristic strip (Theorem 6). Here S^*M denotes the co-sphere or co-tangential sphere bundle of the underlying real analytic manifold M, which we take to be a domain in $\boldsymbol{R}^{n+1}$ containing the origin.

This paper is a summary of a part of forthcoming paper Kawai [7] in which details will be given. Throughout this note P will denote a linear partial differential operator of order m and of simple characteristics with analytic coefficients, whose principal symbol we denote by P_m.

We first state a theorem essentially due to Hamada [1], which generalizes the Cauchy-Kovalevsky theorem.

Theorem 1. *Let P be a partial differential operator with holomorphic coefficients defined near the origin of $\boldsymbol{C}^{n+1}$. (Hereafter we denote a point in C^{n+1} by $(t,z)=(t,z_1,\cdots,z_n)$ and assume $P_m(t,z;1,0)\neq 0$ near the origin.) We assume that the solutions $\tau=\tau_j(t,z;\xi)$ $(j=1,\cdots,m)$ of the equation $P_m(t,z;\tau,\xi)=0$ are mutually disjoint near $(t,z;\xi)=(0,0;\xi_0)$ and consider the following singular Cauchy problem:*

$$\text{(SC)}\qquad \begin{cases} P(t,z,\partial/\partial t,\partial/\partial z)u_k(t,z,y;\xi)=0 \\ (\partial/\partial t)^j u_k(t,z,y;\xi)|_{t=0}=\delta_{jk}(1/<z-y,\xi>)^n \end{cases}$$
$$(0\leqq j,k\leqq m-1,|\xi|=|\xi_0|=1,|\xi-\xi_0|\ll 1).$$

Then (SC) *admits a unique local solution $u_k(t,z,y;\xi)$ which is a multivalued analytic function of $(t,z)\in\boldsymbol{C}^{n+1}$ defined outside $K^{(1)}(y,\xi)\cup\cdots\cup K^{(m)}(y,\xi)$; here $K^{(l)}(y,\xi)=\{(t,z)\,|\,\varphi^{(l)}(t,z,y;\xi)=0\}$, $l=1,\cdots,m$, denote the m (non-singular) characteristic surfaces of P_m passing through the intersection of complex hypersurfaces $t=0$ and $\langle z,\xi\rangle=\langle y,\xi\rangle$ in $\boldsymbol{C}^{n+1}$. $\varphi^{(l)}(t,z,y;\xi)$ denotes the corresponding characteristic function or the phase function satisfying $P_m(t,z;\mathrm{grad}_{(t,z)}\varphi^{(l)}(t,z,y;\xi))\equiv 0$. Furthermore u has the form $\sum_{l=1}^m u^{(l)}$, where the summand $u^{(l)}$*