207. *On Dirichlet Series whose Coefficients are Class Numbers of Integral Binary Cubic Forms*

By Takuro SHINTANI

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1. In this note we give a concrete example of "zeta functions associated with prehomogeneous vector spaces" introduced by Professor M. Sato.

2. We denote by V the vector space of real binary cubic forms. For every $x=(x_1, x_2, x_3, x_4) \in \boldsymbol{R}^4$, we define $F_x \in V$ as follows:

$$F_x(u, v)=x_1u^3+x_2u^2v+x_3uv^2+x_4v^3.$$

In the following, we identify V with $\boldsymbol{R}^4$ by the linear isomorphism: $x \to F_x$. V becomes a $GL(2, \boldsymbol{R})$-module if we put

$$g \cdot F_x((u, v))=F_x((u, v)g)=F_{g \cdot x}((u, v)) \qquad (g \in GL(2, \boldsymbol{R})).$$

For every $x \in V$, we denote by $P(x)$ the discriminant of F_x. We have

$$P(x)=x_2^2x_3^2+18x_1x_2x_3x_4-4x_1x_3^3-4x_2^3x_4-27x_1^2x_4^2$$

and

$$P(g \cdot x)=(\det g)^6P(x) \qquad (g \in GL(2, \boldsymbol{R})).$$

In the following, we put

$$\chi(g)=(\det g)^6 \qquad (g \in GL(2, \boldsymbol{R})).$$

For every $x, y \in V$, we put

$$\langle x, y\rangle=x_4y_1-\frac{1}{3}x_3y_2+\frac{1}{3}x_2y_3-x_1y_4.$$

We denote by $\mathcal{S}(V)$ the space of rapidly decreasing functions on V and define the Fourier transform $\hat{f}$ of $f \in \mathcal{S}(V)$ as follows:

$$\hat{f}(x)=\int_V e^{2\pi i\langle x, y\rangle}f(y)dy.$$

3. We denote by L the lattice of integral binary cubic forms. We have

$$L=\{F_x\,;\, x \in \boldsymbol{Z}^4\}.$$

Then L is invariant under the action of the $SL(2, \boldsymbol{Z})$. Two elements x, y of L are said to be equivalent if there exists a $\gamma \in SL(2, \boldsymbol{Z})$ such that $x=\gamma \cdot y$.

For every integer $m \neq 0$, we denote by L_m the set of integral binary cubic forms whose discriminants are m. It is known that there exist only finite number of equivalence classes in L_m. We denote by $h(m)$ the number of equivalence classes in L_m. Let

$$x_1, \cdots, x_{h(m)}$$

be the representatives of equivalence classes in L_m. When $m<0$,