14. *A Simple Geometric Construction of Weakly Mixing Flows which are not Strongly Mixing*

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1. The existence of measure preserving transformations which are weakly but not strongly mixing has been discussed by Halmos [4], Kakutani-von Neumann [5] and Chacon [1], [2], [3]. Maruyama [6] has shown the existence of Gaussian flows of this type by some results in Gaussian processes. In this short paper we shall give a general method for constructing flows of the type, of which idea is obtained from Chacon [2], [3].

2. Let $(\Omega, \mathcal{L}, \mu)$ be a Lebesgue space, where $\Omega=[0,1)\times[0,1)$, $\mathcal{L}$ is the product Lebesgue class and μ is the usual product Lebesgue measure defined on $\mathcal{L}$.

Definition 1. A flow $\{T_t\}$ on $(\Omega, \mathcal{L}, \mu)$ is said to be ergodic if there exists a positive number t such that $\mu(T_tA\cap B)>0$ holds for every pair A, B from $\mathcal{L}$ with positive measure.

Definition 2. If there exist a complex number λ with the absolute value one and a function f in $L^2(\Omega)$ such that

$$f(T_t(x,y))=\lambda^t f(x,y) \qquad \text{for a.a. } (x,y)\in\Omega \text{ and all } t,$$

we call λ and f an eigenvalue and an eigenfunction corresponding to λ, respectively.

Definition 3. A flow $\{T_t\}$ is weakly mixing if the flow cannot have simple eigenvalues other than one.

Definition 4. A flow $\{T_t\}$ is strongly mixing if

$$\lim_{t\to\infty}\mu(T_tA\cap B)=\mu(A)\mu(B)$$

holds for every pair A, B from $\mathcal{L}$ with positive measure.

Definition 5. For a set A of $\mathcal{L}$ with positive measure, a local flow φ_t on A is defined as follows:

$$\varphi_t(x,y)=\begin{cases}(x, y+t) & \text{if } (x,y+t)\in A,\\ \text{undefined elsewhere,}\end{cases}$$

for each $(x,y)\in A$.

Our main result may be stated as follows:

Theorem. *There exists a weakly mixing flow $\{T_t\}$ on $(\Omega, \mathcal{L}, u)$ which is not strongly mixing.*

Proof. After the flow is constructed, we will prove that it is weakly but not strongly mixing using a direct argument. The first step