9. *On H-closedness and the Wallman H-closed Extensions. II*[*)]

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(Comm. by Kinjirô KUNUGI, M. J. A., Jan. 12, 1971)

4. The Wallman H-closed extensions. Let X be a space, $\mathfrak{C}$ the family of all closed subsets of X, and $W(X)$ the collection of all subfamilies of $\mathfrak{C}$ which possess the PFIP and are maximal in $\mathfrak{C}$ relative to this property. Two elements w_1, w_2 of $W(X)$ are said to be equivalent if both of them contain the closures of the neighborhoods of the same point x in X. An equivalent class in $W(X)$ corresponding to a point x is called a fixed end and denoted by $\mathfrak{A}(x)$; an element in $W(X)$ which does not belong to any fixed end is called a free end and denoted by $\mathfrak{A}$. We denote by $\omega(X)$ the collection of all fixed and free ends in X. For an open subset U of X let $U^*=\{\mathfrak{A}(x)\,;\, x\in U\}$. We introduce the following topology for $\omega(X)$, called Katětov topology: the neighborhoods for fixed ends $\mathfrak{A}(x)$ are U^* if $x\in U$ and for free ends $\mathfrak{A}$ are $U^*U\ \{\mathfrak{A}\}$, where U is the interior of a closed set A belonging to $\mathfrak{A}$. The space $\omega(X)$ with Katětov topology is H-closed and the subspace consisting of all $\mathfrak{A}(x)$ is homeomorphic to X (also denoted by X). Moreover, the H-closed space $\omega(X)$ has the following properties: (1) X is dense in $\omega(X)$, (2) X is open in $\omega(X)$, and (3) $\omega(X)$-X is discrete (see [5]).

Lemma 5. *Every bounded real-valued continuous function f on X can be continuously extended over $\omega(X)$.*

Proof. Suppose that f can not be continuously extended at $\mathfrak{A}\in\omega(X)$. Then there is an $\varepsilon>0$ such that to the interior U of each member A of $\mathfrak{A}$ there are $x, y\in\bar{U}$ satisfying the condition $f(y)-f(x)>\varepsilon$. It is clear that for two members A_α, A_β of $\mathfrak{A}$ $f(y_\beta)-f(x_\alpha)>\varepsilon$, since $A_\alpha\cap A_\beta=A_{\alpha\beta}$, $f(y_{\alpha\beta})\le\min.\{f(y_\alpha), f(y_\beta)\}$, $f(x_{\alpha\beta})\ge\max.\{f(x_\alpha), f(x_\beta)\}$, and $f(y_{\alpha\beta})-f(x_{\alpha\beta})>\varepsilon$. Let L be the least upper bound of $\{f(x_\alpha)\}$ and M the greatest lower bound of $\{f(y_\alpha)\}$. Then $M>L$ and $M-L\ge\varepsilon$. If $P=\left\{x\,;\, f(x)\ge M-\dfrac{\varepsilon}{3}\right\}$ and $Q=\left\{x\,;\, f(x)\le L+\dfrac{\varepsilon}{3}\right\}$, then both P and Q intersect each member of $\mathfrak{A}$ in sets containing non-vacuous open sets and belong to $\mathfrak{A}$. But $P\cap Q=\emptyset$ and the contradiction proves the lemma.

Corollary. *Every unbounded real-valued continuous function on*

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*) Presented to the Amer. Math. Soc., Aug., 28 (1970).
The work was done during Sabbatical Leave, Spring (1970).