# 39. *Some Properties of wM-Spaces*

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**1.** Recently the notion of $wM$-spaces has been introduced by one of the authors (cf. T. Ishii [7]), which is a generalization of the notion of $M$-spaces due to K. Morita [12]. The purpose of this paper is to state further properties of $wM$-spaces, most of which are concerned with metrization of $wM$-spaces. A topological space $X$ is called a $wM$-space if there exists a sequence $\{\mathfrak{U}_n\}$ of open coverings of $X$ satisfying the condition below:

$$(\mathrm{M}_2)\begin{cases}\text{If } \{K_n\} \text{ is a decreasing sequence of non-empty subsets of } X \text{ such} \\ \text{that } K_n \subset \mathrm{St}^2(x_0, \mathfrak{U}_n) \text{ for each } n \text{ and for some fixed point } x_0 \text{ of } X, \\ \text{then } \cap \bar{K}_n \neq \emptyset.\end{cases}$$

In the above definition, we may assume without loss of generality that $\{\mathfrak{U}_n\}$ is decreasing.

Now we shall state a remarkable property of $wM$-spaces, which was essentially proved in [7, I, Theorem 2.4].

**Theorem 1.1.** *Let $X$ be a $wM$-space, and $\{F_\lambda\}$ a locally finite collection of closed subsets of $X$. Then there exists a locally finite collection $\{H_\lambda\}$ of open subsets of $X$ such that $F_\lambda \subset H_\lambda$ for each $\lambda$.*

This theorem will play an important role in the proof of the following theorems. For topological spaces no separation axiom is assumed unless otherwise specified.

**2.** First we shall prove the following

**Theorem 2.1.** *Every point-finite open covering of a $wM$-space has a locally finite open refinement.*

This theorem is an immediate consequence of the following lemmas.

**Lemma 2.2.** *Every point-finite open covering of a $wM$-space has a $\sigma$-locally finite open refinement.*

With the aid of Theorem 1.1, we can prove this lemma by the similar method as in the case of a collectionwise normal space (cf. E. Michael [9], K. Nagami [14]), and hence we omit the proof.

**Lemma 2.3** ([7, I, Lemm 2.6]). *Every $wM$-space is countably paracompact.*

**3.** Recently the interesting properties of semi-stratifiable spaces has been obtained by G. D. Creede [3] and Ja. A. Kofner [8].

**Theorem 3.1.** *A space $X$ is metrizable if and only if $X$ is a $T_2$, semi-stratifiable $wM$-space.*