34. *Construction of a Local Elementary Solution for Linear Partial Differential Operators. II*

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(Comm. by Kunihiko KODAIRA, M. J. A., Feb. 12, 1971)

Let $P(x, D_x)$ be a linear partial differential operator with real analytic coefficients defined on a domain containing the origin in $\boldsymbol{R}^n$. We denote its principal symbol by $P_m(x, \xi)$. Assume that $P(x, D_x)$ has simple characteristics, that is, $\mathrm{grad}_\xi P_m(x, \xi) \neq 0$ whenever $P_m(x, \xi)=0$.

In this note we first construct a local elementary solution for P under the condition (P), which is concerned with the behaviour of the characteristic surfaces. Secondly we prove that the condition (P) follows from the condition $(\mathrm{NT})_f$, which is deeply related with the work of Nirenberg and Treves [6], [7]. The condition $(\mathrm{NT})_f$ does not cover all the possibilities of the solvable partial differential operators in the theory of hyperfunctions. Thus our result is weaker than that of Nirenberg and Treves [7] concerning distribution solutions. Our analysis is different from theirs in the point that we treat the problem in the framework of hyperfunctions or rather in that of Sato's sheaf $\mathcal{C}$ defined on the cotangential sphere bundle (or co-sphere bundle in short). For the notion of the sheaf $\mathcal{C}$ we refer the reader to Sato [8], [9]. We hope, however, our method of construction of an elementary solution given in Theorem 2 reveals the geometrical meaning of condition $(\mathrm{NT})_f$.

In Theorem 4 and Theorem 5 we also treat two cases which are not covered by condition $(\mathrm{NT})_f$. We remark that the three features, which appear in Theorems 2, 4 and 5 respectively, are typical ones about the behaviour of the characteristic surfaces.

We have constructed a local elementary solution $E(x, y)$ for a linear partial differential operator P with simple characteristics and with real coefficients in its principal symbol and investigated its singularities in our previous note [4], so that in the sequel we consider the case where the principal symbol $P_m(x, \xi)$ of P has the form $A_m(x, \xi)+iB_m(x, \xi)$ where A_m and B_m are real and $B_m \not\equiv 0$. We can assume that $\mathrm{grad}_\xi A_m \neq 0$ when $P_m=0$ without the loss of generalities. The details of this note will be published elsewhere. (See also Kawai [5].)

Our method of construction of an elementary solution for P is just the same as that employed in our previous note [4]. We first repeat the fundamental theorem essentially due to Hamada [1] in a form which is suitable for the present situations.