66. *Uniformities for Function Spaces and Continuity Conditions*

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1. Introduction. Let $\mathfrak{C}$ be, for example, a semi-group of continuous mappings of a uniform space X into itself. For given set-entourage uniformities on $\mathfrak{C}$ a number of properties have been studied. In the other way, we take the following basic criterion on the uniformities for $\mathfrak{C}$ in order to find out new natural uniformities on $\mathfrak{C}$ if possible:

1) the mapping $(u, x)\to u(x)$ of $\mathfrak{C}\times X$ into X is continuous,

2) the mapping $(u, v)\to uv$ of $\mathfrak{C}\times\mathfrak{C}$ into $\mathfrak{C}$ is continuous.

Under this view we get Theorems 2, 3, 4, and 5 in the present paper, and then apply some of the results to get Theorems 6 and 7.

Proofs are omitted, most of which are straightforward. For terms and notations we follow Bourbaki [2].

2. Uniformizability condition.

Theorem 1. *Let X be a set; let Y be a set endowed with a uniform structure $\mathfrak{U}$ which is not the coarsest; let $\mathfrak{S}$ be a family of subsets of X; and let $\mathfrak{F}$ be the family of all mappings of X into Y. For each $A\in\mathfrak{S}$ and each $U\in\mathfrak{U}$, let $W(A, U)$ denote the set of all pairs (u, v) of mappings of X into Y such that $(u(x), v(x))\in U$ for all $x\in A$. Then, as A runs through $\mathfrak{S}$ and U runs through $\mathfrak{U}$, the sets $W(A, U)$ form a fundamental system of entourages of a uniformity on $\mathfrak{F}$ if and only if for any two sets $A_1, A_2\in\mathfrak{S}$ there exists a set $A_3\in\mathfrak{S}$ such that $A_3\supset A_1\cup A_2$.* (1)

Definition 1. For a family $\mathfrak{S}$ that satisfies (1), the uniformity on $\mathfrak{F}$ generated by $\{W(A, U)\mid A\in\mathfrak{S}, U\in\mathfrak{U}\}$ is called *$\mathfrak{S}$-uniformity.*

Proposition 1. *Let $X, Y, \mathfrak{U}$ and $\mathfrak{F}$ be the same as those in Theorem 1. Let $\mathfrak{S}$ be a non-empty family of subsets of X; let $\mathfrak{S}^*$ be the family of all sets that are finite unions of sets belonging to $\mathfrak{S}$; let $\mathfrak{S}^{**}$ be the family of all subsets of sets belonging to $\mathfrak{S}^*$; for each $A\in\mathfrak{S}$ let $\mathfrak{W}_A$ denote the $\{A\}$-uniformity on $\mathfrak{F}$, where $\{A\}$ is the family consisting of the set A only. Let $\mathfrak{W}^*$ and $\mathfrak{W}^{**}$ denote $\mathfrak{S}^*$-, and $\mathfrak{S}^{**}$-uniformity on $\mathfrak{F}$ respectively, and let $\mathfrak{W}$ denote the uniformity of $\mathfrak{S}$-convergence in the sense of Bourbaki* [2]. *Then*

$$\mathfrak{W}=\mathfrak{W}^*=\mathfrak{W}^{**}=\bigcup_{A\in\mathfrak{S}^*}\mathfrak{W}_A=\bigcup_{A\in\mathfrak{S}^{**}}\mathfrak{W}_A.$$

The following propositions are the simple cases where some properties of $\mathfrak{W}$ determine $\mathfrak{S}$ and $\mathfrak{U}$.

Proposition 2. *The following conditions are equivalent:*