122. *Hyperfunction Solutions of the Abstract Cauchy Problem*

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In this note we will report a few results on hyperfunction solutions of the abstract Cauchy problem

$$\begin{cases} \dfrac{du(t)}{dt}=Au(t) \\ u(0)=a \end{cases}$$

where A is a closed linear operator in a Banach space X and $a \in X$. We discuss conditions for existence, uniqueness and regularity.

Hyperfunctions defined as boundary values of holomorphic functions are more general than Schwartz distributions. Hence if the Cauchy problem is well-posed in the sense of distribution, then we shall find that it is also well-posed in the sense of hyperfunction.

Distribution solutions were investigated by J. Chazarain [1], G. Da Prato, U. Mosco [2], D. Fujiwara [3], J. L. Lions [4] and T. Ushijima [7].

A complete proof will be published elsewhere.

§1. Hyperfunctions with values in a Banach space. We shall use vector valued hyperfunctions of one variable. The case of one variable is much simpler than that of many variables. We refer the reader to M. Sato [6] for the scalar case.

Let E be a Banach space. Consider the space $\mathcal{O}(\Omega, E)$ of all E-valued holomorphic functions defined on Ω, where Ω is an open subset in $\boldsymbol{C}^1$. Let S be an open set in $\boldsymbol{R}^1$. We define an E-valued hyperfunction to be an element of the quotient space:

$$\mathcal{B}(S,F)=\frac{\mathcal{O}(D-S,E)}{\mathcal{O}(D,E)},$$

where D is a complex neighbourhood of S, which contains S as a closed set.

For E-valued hyperfunctions we can establish results similar to the case of scalar hyperfunctions. In particular $\mathcal{B}(S,E)$ does not depend on the complex neighbourhood D of S. The E-valued hyperfunctions are flabby, that is, for every $f \in \mathcal{B}(S,E)$ there exists $\mathcal{F} \in \mathcal{B}(R^1,E)$ such that the restriction $\mathcal{F}|_S$ of $\mathcal{F}$ to S coincides with f. The notion of support can be defined. These facts can be proved in a way analogous to M. Sato [6] with the aid of Runge's approximation theorem of E-