117. *Modules over Bounded Dedekind Prime Rings. II*

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This paper is a continuation of [3]. Let D be an s-local domain which is a principal ideal ring. Then every right (left) ideal is an ideal and every ideal of D is a power of $J(D)$(see [2]). We put $J(D)=p_0D=Dp_0$. Then every non-unit $d\in D$ can be uniquely expressed as $d=p_0^k\varepsilon=\varepsilon' p_0^k$, where ε, ε' are units of D and k is an integer.

Let M be a D-module. An element x in M has *height* n if x is divisible by p_0^n but not by p_0^{n+1}; it has *infinite height* if it is divisible by p_0^n for every n. We write $h(x)$ for the height of x; thus $h(x)$ is a (non-negative) integer or the symbol ∞. Terminology and notation will be taken from [3].

Lemma 1. *Let D be an s-local domain which is a principal ideal ring, let M be a D-module and let S be a submodule with no elements of infinite height. Suppose that the elements of order $J(D)$ in S have the same height in S as in M. Then S is pure.*

Lemma 2. *Let D be an s-local domain which is a principal ideal ring and let M be a D-module. Suppose that all elements of order $J(D)$ in M have infinite height. Then M is divisible.*

An R-module is said to be *reduced* if it has no non-zero divisible submodules.

Theorem 1. *Let R be a bounded Dedekind prime ring and let P be a prime ideal of R. If M is a P-primary reduced R-module, then M possesses a direct summand which is isomorphic to eR/eP^n, where e is a uniform idempotent contained in R_P.*

By Theorem 1, we have

Theorem 2. *Let R be a bounded Dedekind prime ring. Then*

(i) *An finitely generated indecomposable R-module cannot be mixed and is not divisible, i.e., it is either torsion-free or torsion. In the former case, it is isomorphic to a uniform right ideal of R and in the latter case, it is isomorphic to eR/eP^n for some prime ideal P, where e is a uniform idempotent contained in R_P.*

(ii) *An indecomposable torsion R-module is either of type P^∞ or isomorphic to eR/eP^n for some prime ideal P, where e is a uniform idempotent contained in R_P.*

Lemma 3. *Let D be an s-local ring with $J(D)=p_0D$ which is a principal ideal domain. Let M be a D-module, let H be a pure submodule*