149. *Commutative Semigroups with Greatest Group-Homomorphism*

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(Comm. by Kenjiro SHODA, M. J. A., Oct. 12, 1971)

§1. **Introduction.** Let S be a commutative semigroup throughout this paper. A homomorphism f of S is called a group-homomorphism of S if $f(S)$ is a group. A congruence ρ on S is called a group-congruence on S if S/ρ is a group. The smallest group-congruence ρ_0 on S is defined to be a congruence on S which is contained in all group-congruences on S. The greatest group-homomorphism $f_0: S \to G_0$ is defined to be a group-homomorphism of S onto a group G_0 having the property that for a given group-homomorphism $f: S \to G$ there is a homomorphism $h: G_0 \to G$ such that $f(x) = hf_0(x)$ for all $x \in S$. In other words $f_0: S \to G_0$ has the so-called universal repelling property with respect to homomorphisms from S onto abelian groups. The natural homomorphism $S \to S/\rho_0$ is a greatest group-homomorphism. The group G_0 is called a greatest group-homomorphic image of S, and G_0 is uniquely determined up to isomorphism. On the other hand there is a homomorphism g_1 of S into an abelian group G_1 having the universal repelling property with respect to homomorphisms from S into abelian groups. The g_1 is called a Grothendieck homomorphism (gr-homomorphism) and G_1 is called a Grothendieck group (gr-group) of S [1], [7], or a free group over S [3, Section 12.1]. For every S, g_1 always exists but f_0 does not in general. The following questions are proposed:

Under what condition on S does there exist f_0?

What structure does S have if f_0 exists?

Group-homomorphisms and group-congruences were studied by many mathematicians, Croisot [4], Dubreil [5], Levi [8], [9], Stoll [11], (also see [3]), while semigroups admitting greatest group-homomorphism have not been so much done except quite recent results by Head [6], McAlister and O'Carroll [10]. However their approach is not near to determining the structure of such commutative semigroups. The purpose of this note is to report our results to characterize commutative semigroups admitting greatest group-homomorphism in terms of (i) various kinds of homomorphisms including gr-homomorphism, (ii) conditions with respect to multiplicative structure. Theorem 3.1 gives (i) and Theorem 4.1 gives (ii). Though the complete construction of the semigroups is still open, Theorem 4.1 and 5.3 contribute to the