171. A New Characterization of Real Analytic Functions

By Akira KANEKO*)
Department of Mathematics, Faculty of Science,
University of Tokyo

(Comm. by Kunihiko KODAIRA, M. J. A., Dec. 13, 1971)

In [1], [2] we have shown that the local operators, the differential operators of infinite order in the theory of hyperfunctions, characterize the hyperfunctions in terms of measures. Here we present a theorem which shows that the local operators also characterize the real analytic functions in terms of continuous, functions. We mention the case of one variable. The case of several variables can be proved in the same way when we apply the result of [2] and use the standard defining functions of the hyperfunctions with compact supports of several variables. It will be published in a forthcoming paper [3].

Proposition. *Let $A_J(K)$ be the space of the real analytic functions on the compact interval K with its topology defined by the seminorms $\|u\|_J=\sup_{x\in K}|J(D)u(x)|$, where $J(D)$ runs over the local operators with constant coefficients* (see [1]). *Then $A_J(K)$ is sequentially complete.*

Proof. By Corollary 6 of [1] (and by a trivial consideration) we have the following domination relation between the topologies of $A(K)$:

$$\sigma(A(K), B[K]) \prec A_J(K) \prec A(K),$$

where $A(K)$ denotes its usual DFS topology, and $\sigma(A(K), B[K])$ the weak topology of $A(K)$ under pairing with its dual space $B[K]$ (the space of hyperfunctions with supports in K). Thus by the theorem of Mackey the bounded sets are common in all these spaces. Since $A(K)$ is DFS, its bounded sets are relatively compact. Now take a Cauchy sequence $\{f_n\}$ in $A_J(K)$. Then $\{f_n\}$ is obviously bounded in $A_J(K)$. Thus it is bounded in $A(K)$, hence relatively compact. Therefore there is a subsequence $\{f_{n'}\}$ which converges to some element $f\in A(K)$ in $A(K)$, hence in $A_J(K)$. Since $\{f_n\}$ is a Cauchy sequence, we have proved that $\{f_n\}$ converges to a real analytic function f in $A_J(K)$. q.e.d.

Theorem. *Let f be a hyperfunction on an open interval $I\subset R^1$. Assume that for any local operator J, $J(D)f$ is a continuous function in I. Then f is real analytic in this interval.*

Proof. Since we are concerned with a local property, we only have to prove that f is real analytic in the interior of any compact

---

*) Supported in part by Fujukai Foundation.